Avalanches in complex spin networks and lattices

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1 Introduction

Reversal processes are of great importance for technological applications, e.g., in information processing and memory devices.

Often hysteresis curves with particular properties are required.

In this respect novel “artificial solids”, arrays of nanoscale magnetic particles, quantum cellular automata and integrated functional nanosystems offer challenging possibilities, yet to be understood.

Currently methods are being developed for patterning, defining and measurements of the magnetic properties at nanometer scale.

Much less attention heve been devoted to the theoretical study of these systems.

1.1 Spin Dynamics

A spin, $S = \pm 1$, is associated with each node i on the network. The spins interact along the links connecting neighbouring nodes. We assume uniform antiferromagnetic (AF) interaction $J = -1$ and uniform external field H according to the Hamiltonian

$$\mathcal{H} \equiv -J \sum_{i,j>i} a_{ij} S_i S_j - H \sum_i S_i, \qquad (1)$$

where $J = -1$

$$a_{ij} = \begin{cases} 1 & \iff \text{nodes } i \text{ and } j \text{ are connected,} \\ 0 & \iff \text{elsewhere.} \end{cases}$$

An avalanche represents a fraction of spins that are reversed at current field value in order to minimize the energy within locally available geometry.

2 Complex Networks

The structure is grown by systematic addition and linking of nodes. At each growth step i a node is added and linked to M different nodes selected among $i-1$ preexisting nodes.

Selection of a target node, is given by a specified probability $p(k)$, where k is node degree.

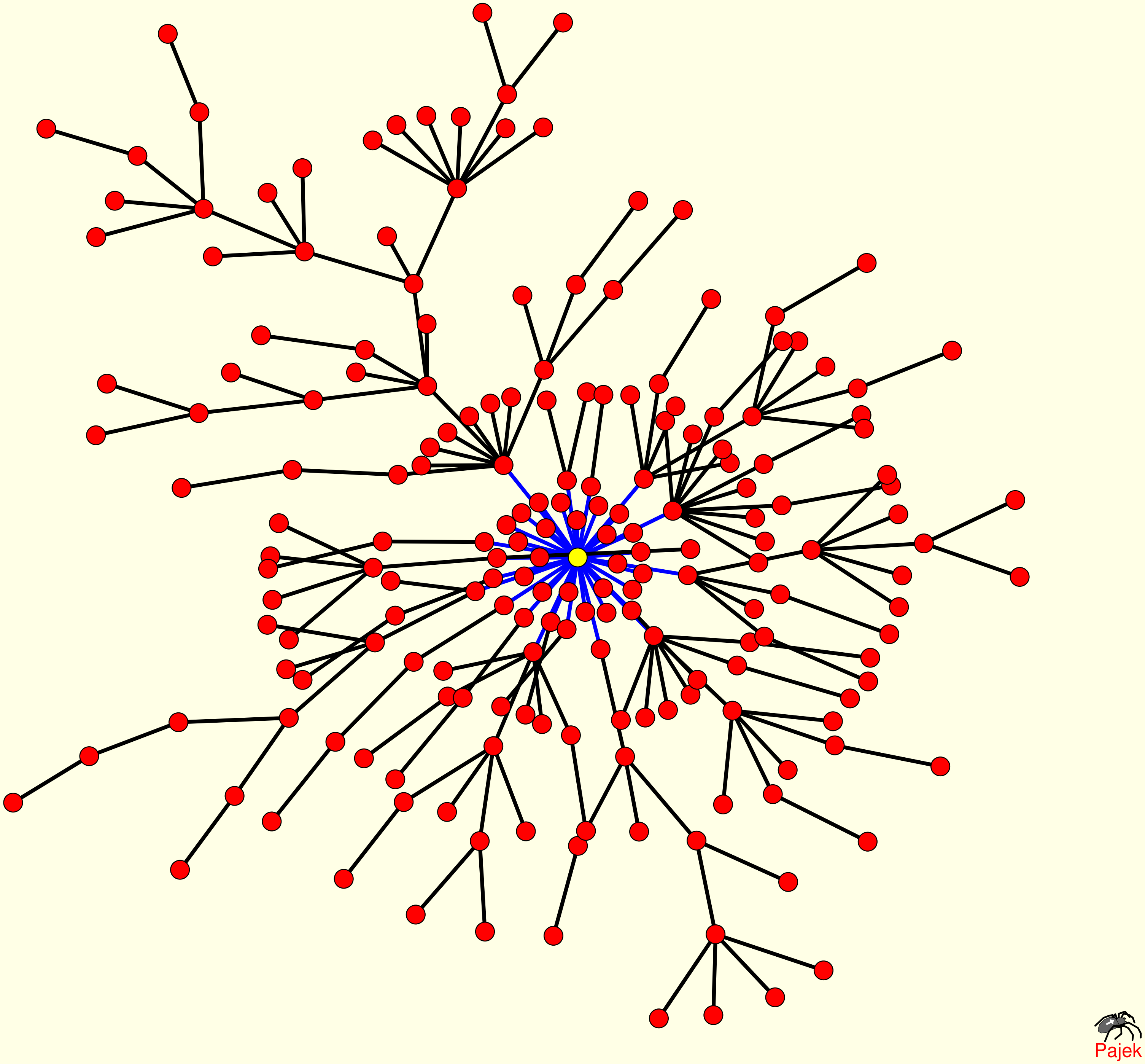
For $p(k) = const = 1/i$, i.e., independent of properties of target nodes, the emergent network is known to have an exponential degree distribution.

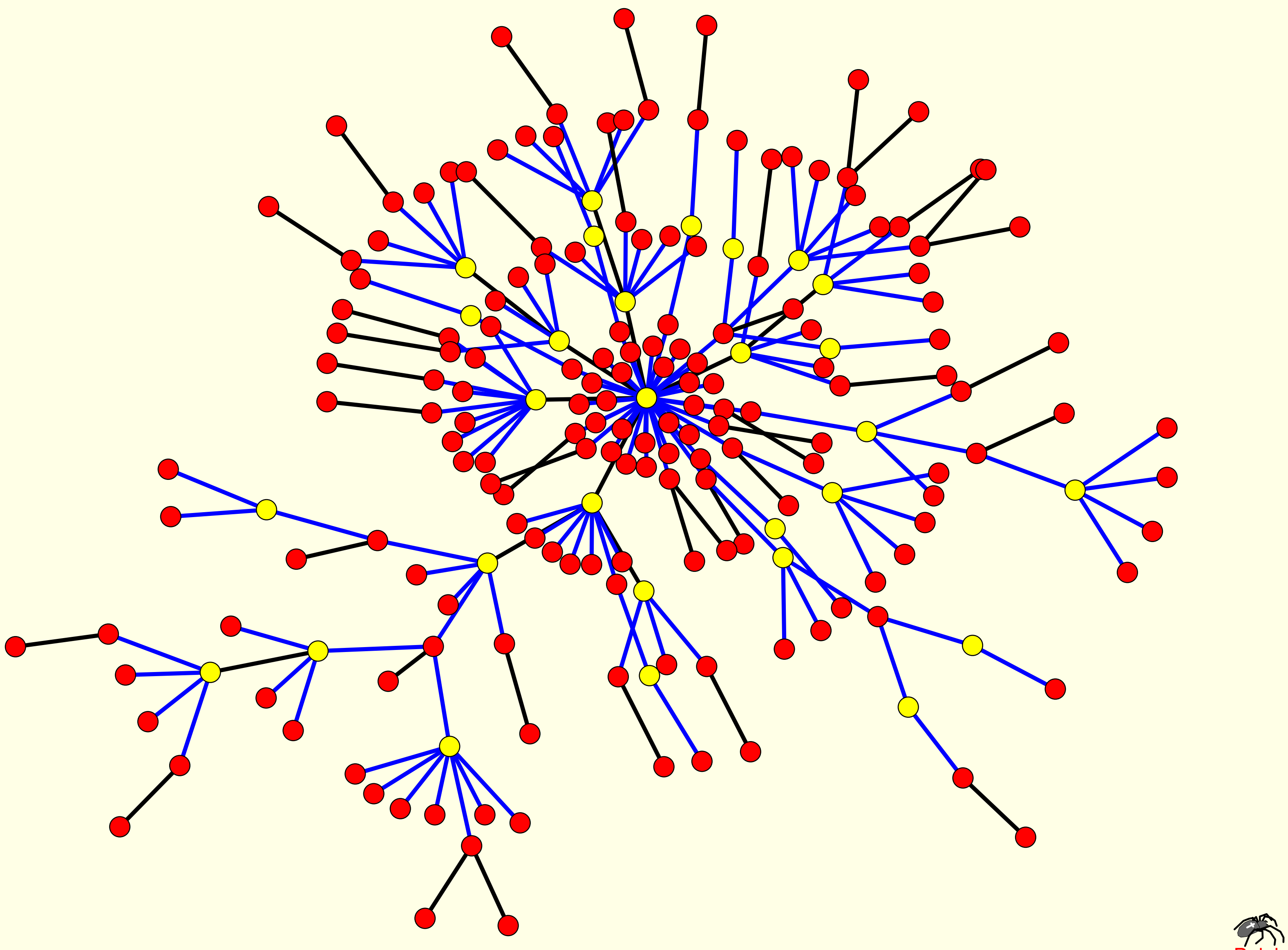
In contrast, the preferential attachment, where the selection probability $p(k) \propto k$, is shown to lead to a scale-free degree distribution.

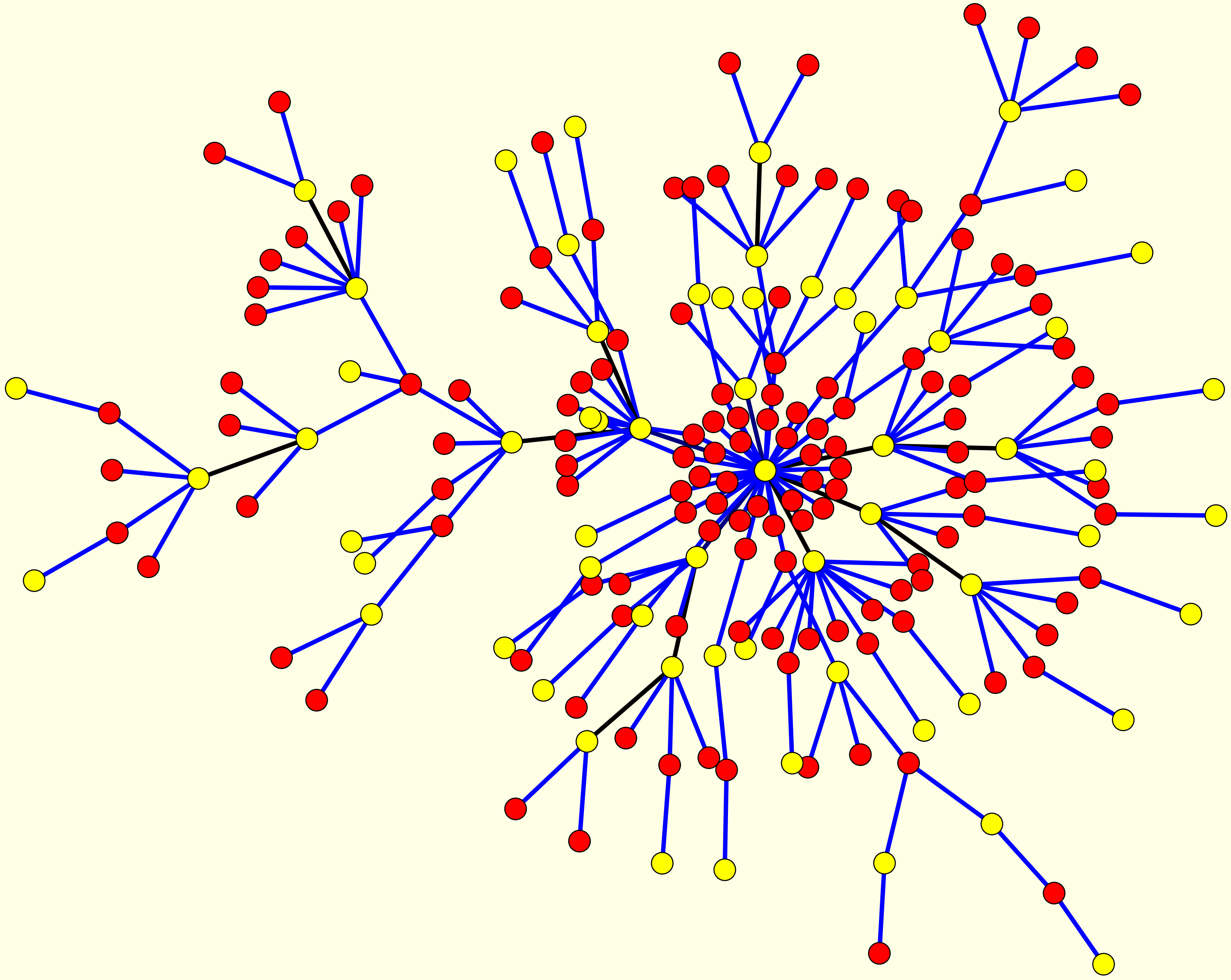
2.1 Network Structure

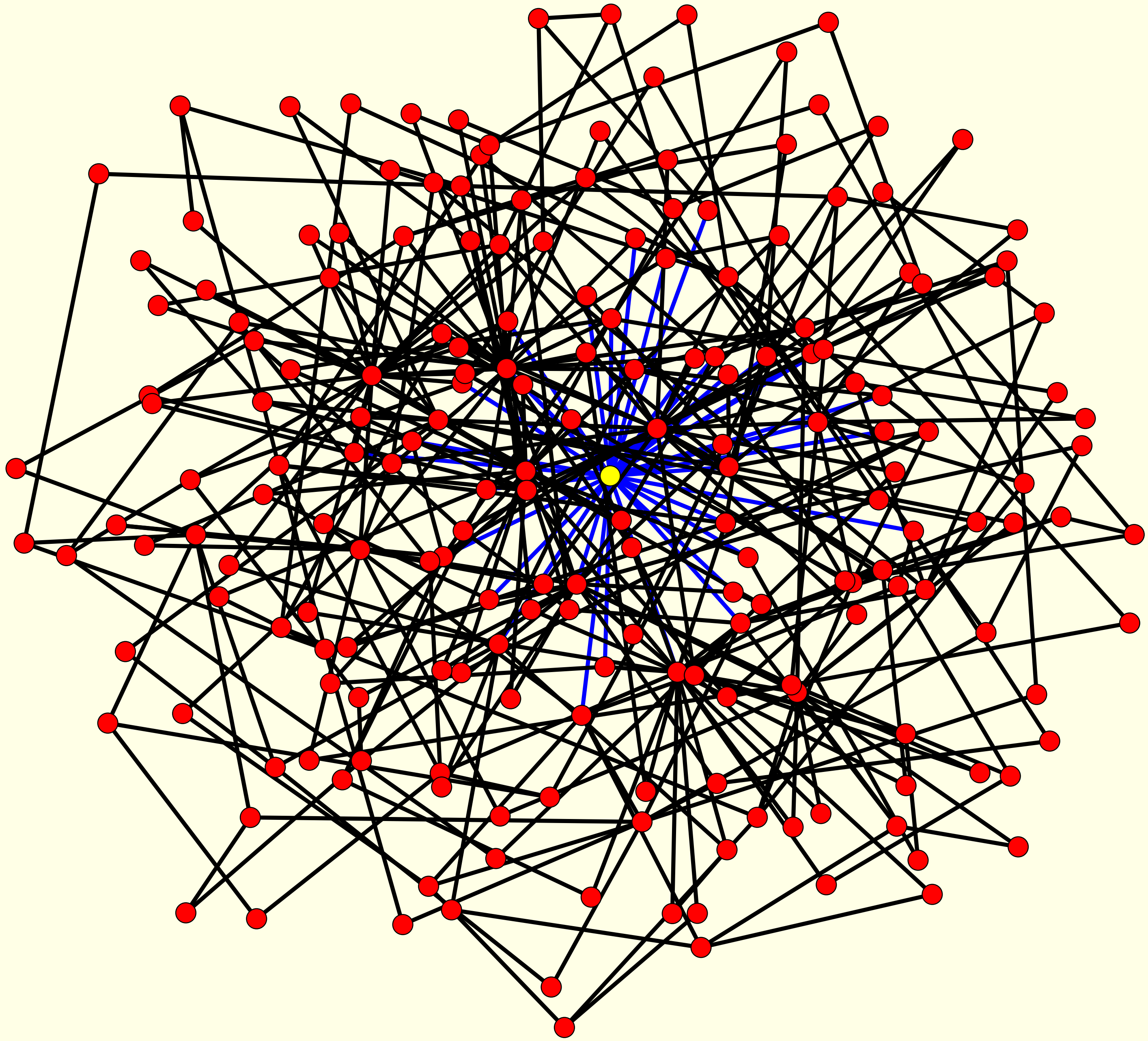
Snapshots from simulation for scale-free tree ($M = 1$) and scale-free simple graph ($M = 2$) for $N = 200$ and magnetic field H soon after the first flip ($H = -H_{\text{max}} + 1 - \delta$), intermediate filed and for $H = 0 - \delta$.

Spins $S_i = -1$ (red) and $S_i = +1$ (yellow) and their pairs $S_i S_j = +1$ (black) $S_i S_j = -1$ (blue) for all $1 \leq i, j \leq N$.


Pajek


Pajek

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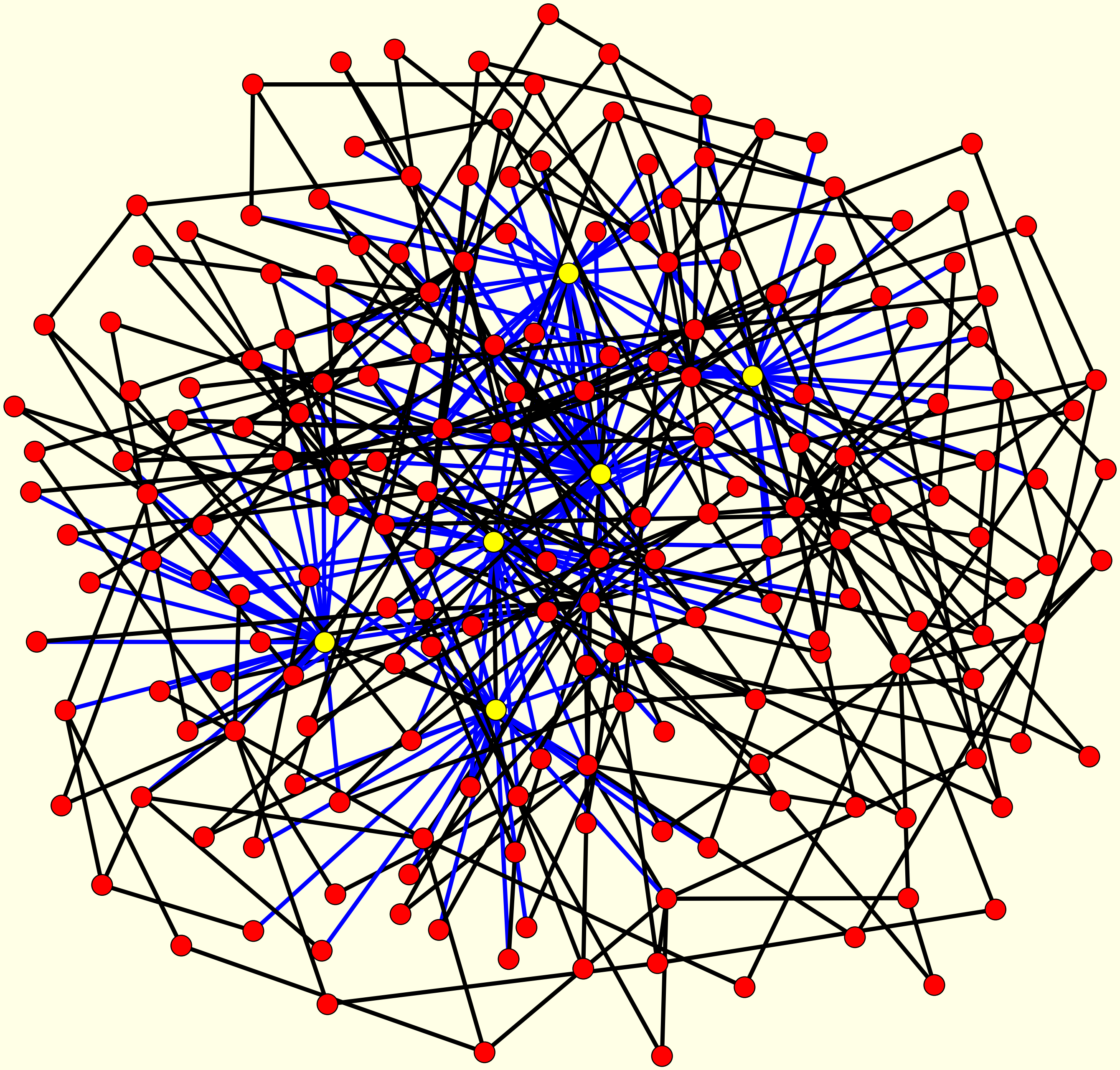
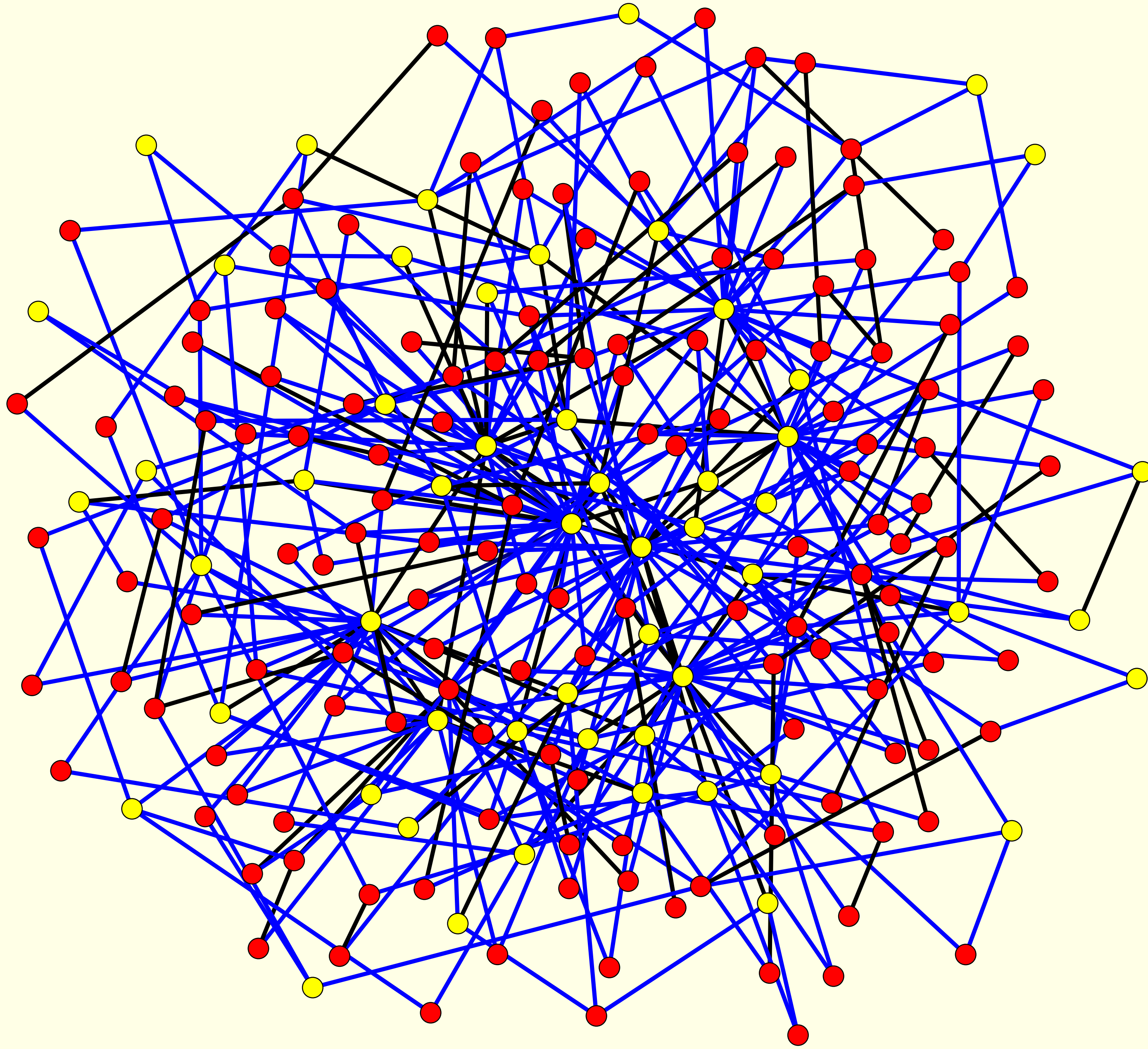




2.2 Histeresis Loops

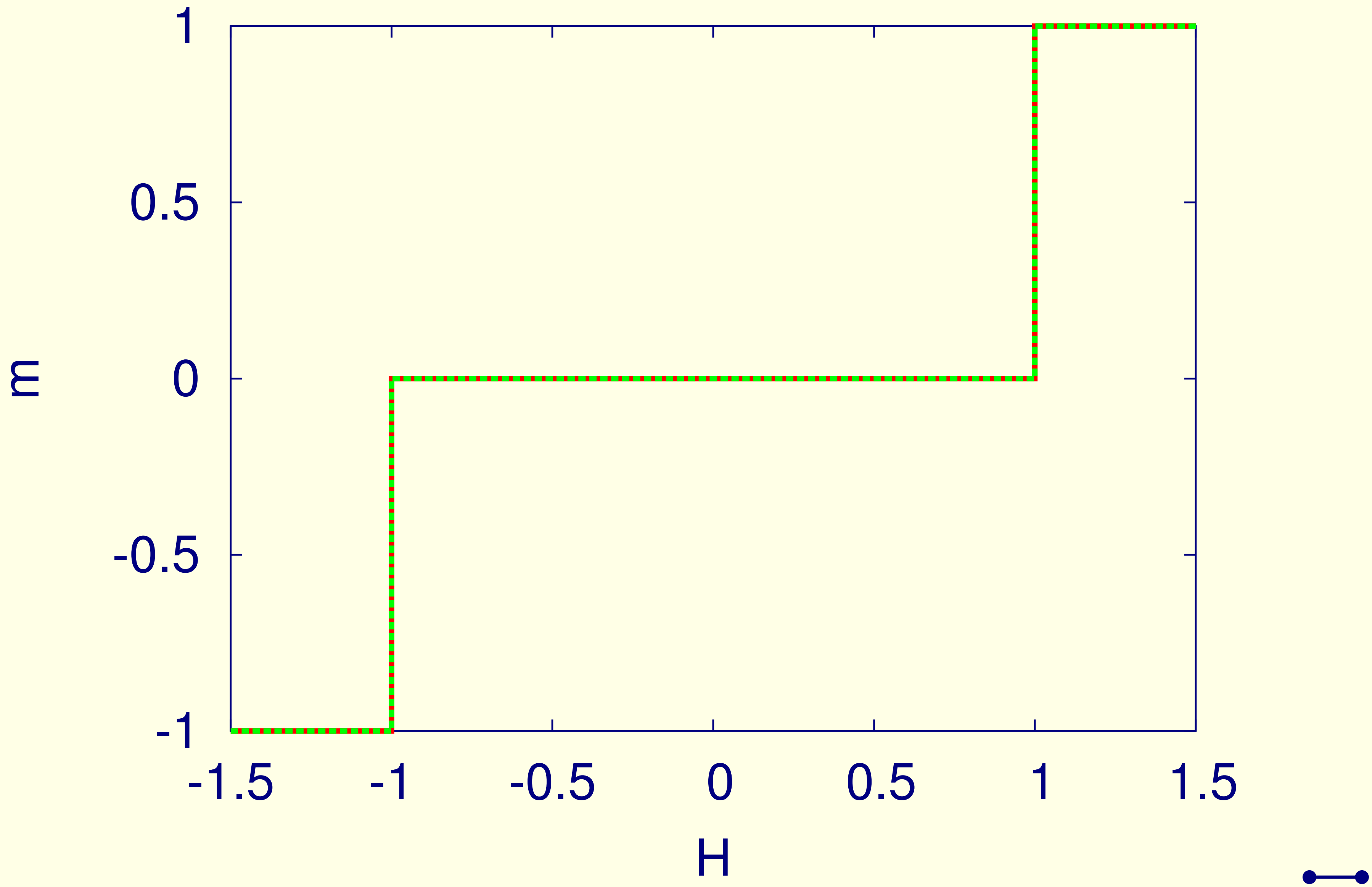
Graphs are placed in saturating magnetic field $H > 0$, which forces all spins to be positive.

We scan the field H by integer steps, from $n+\delta$ to $n-1+\delta$, to omit the values $H = n$ where the spin orientation could be ambiguous.

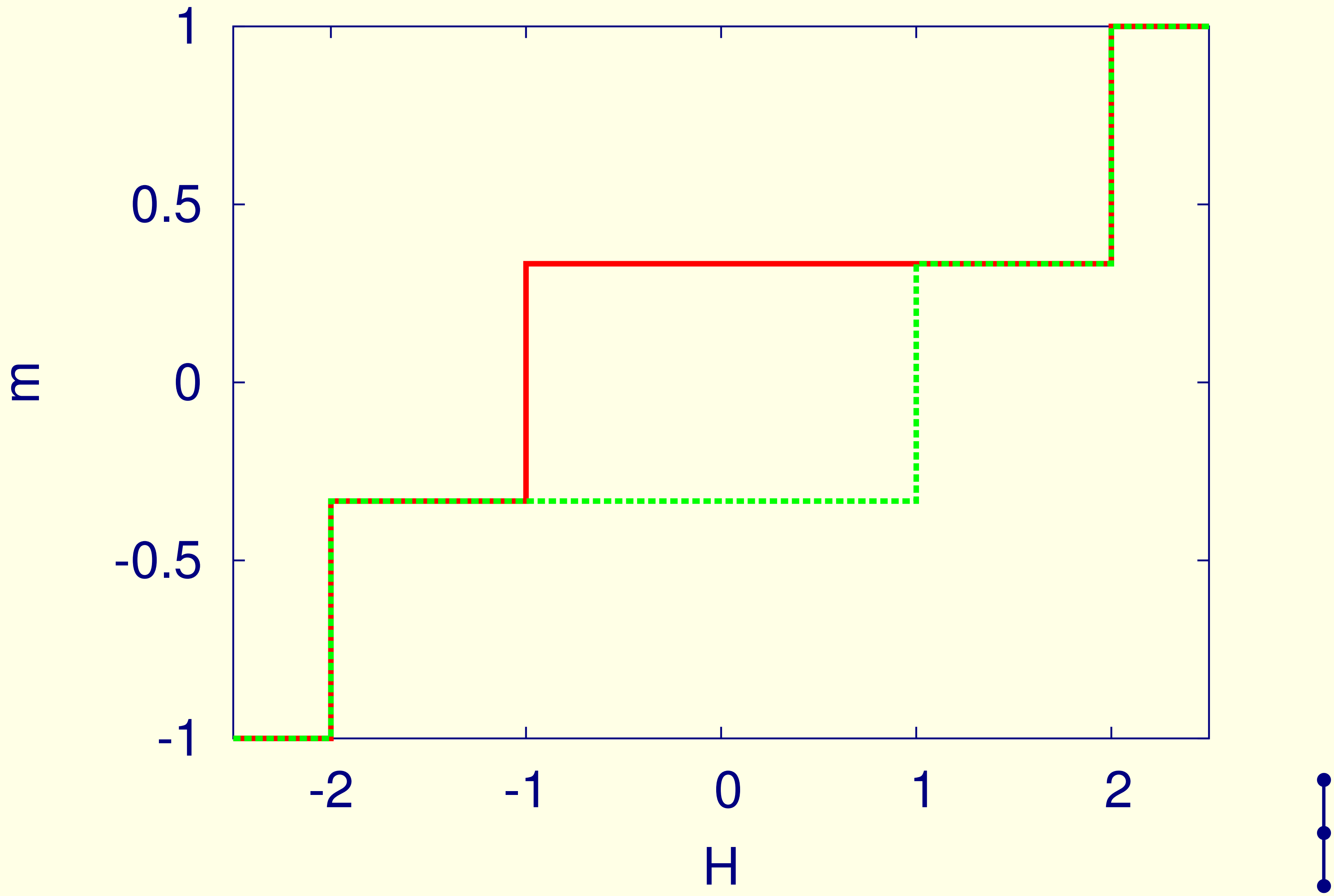
As the field decreases gradually at $T = 0$, some spins are flipped ($S_i \to -S_i$) because of the AF interaction with their neighbors.

2.2.1 Toy Story

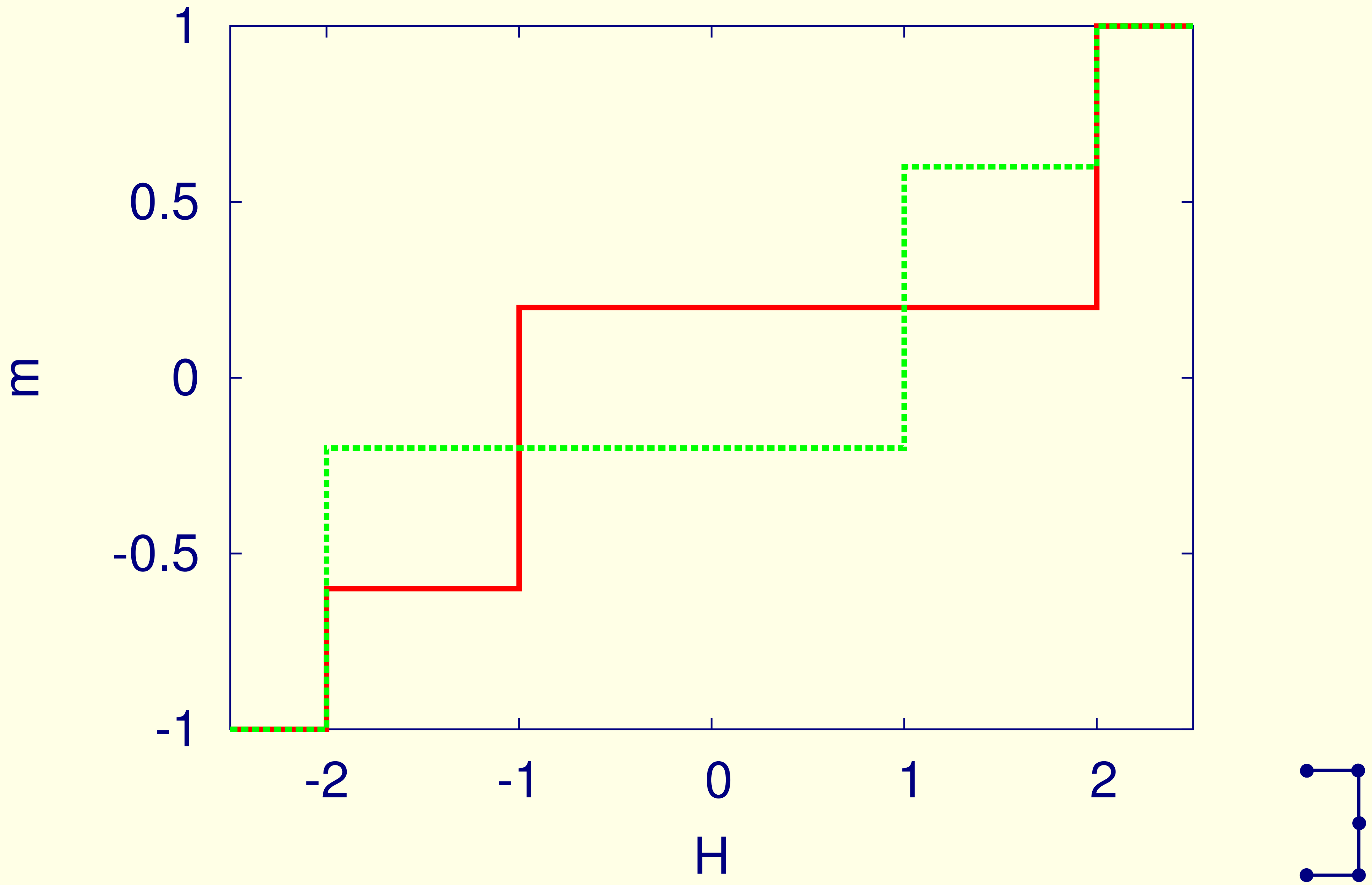
- Toy graphs and their hysteresis loops for decreasing field (red line) and increasing field (green line).
- For some of them, the results depend on the order of updating.
- On the other hand the synchronous scheme of spin updates leads to oscillations: a state when $S_i \longrightarrow -S_i$ becomes permanent.



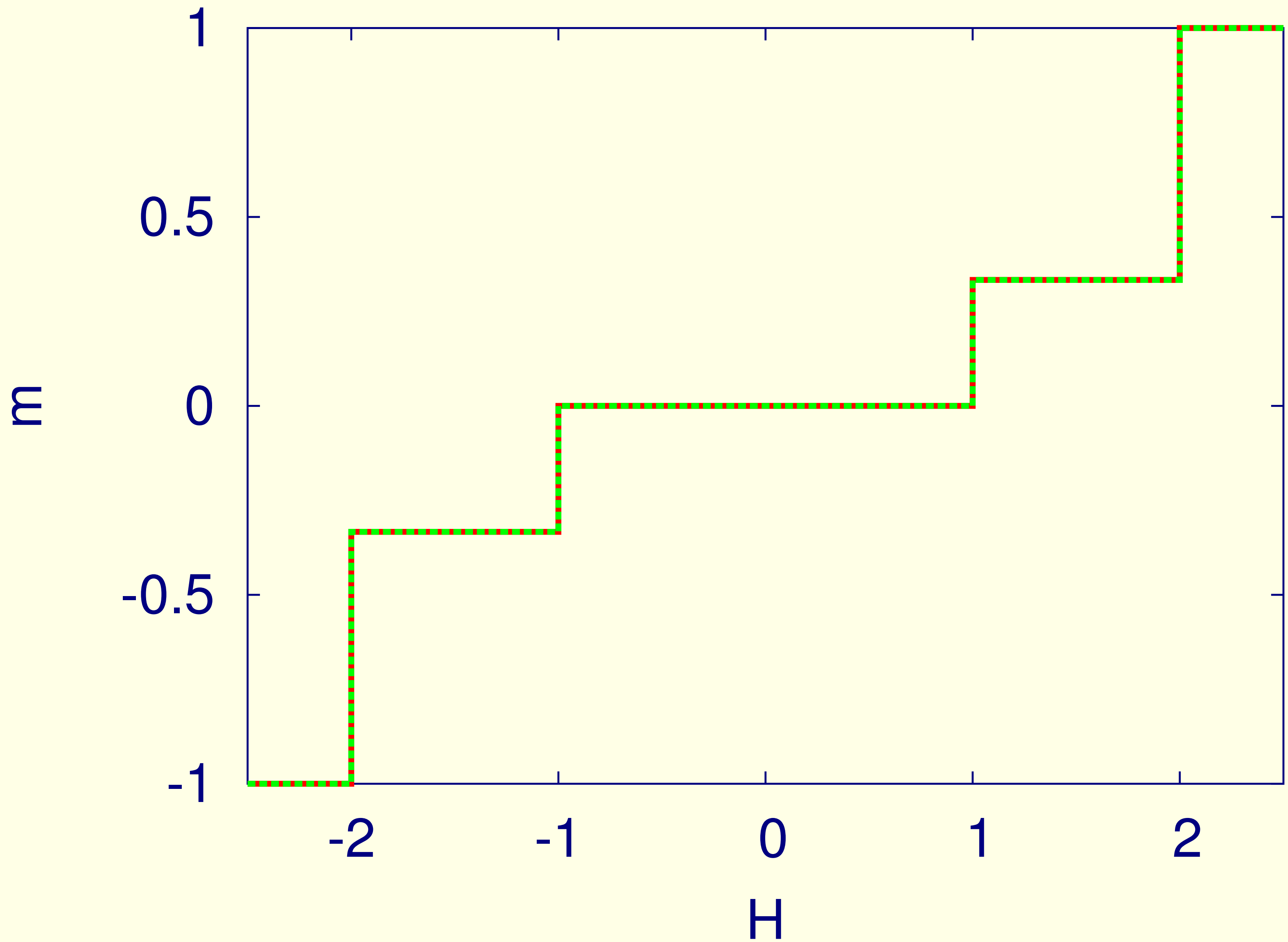
m
H
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5



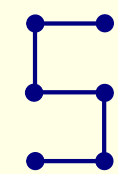
1
0.5
0
-0.5
-1
m
-2
-1
0
1
2
H

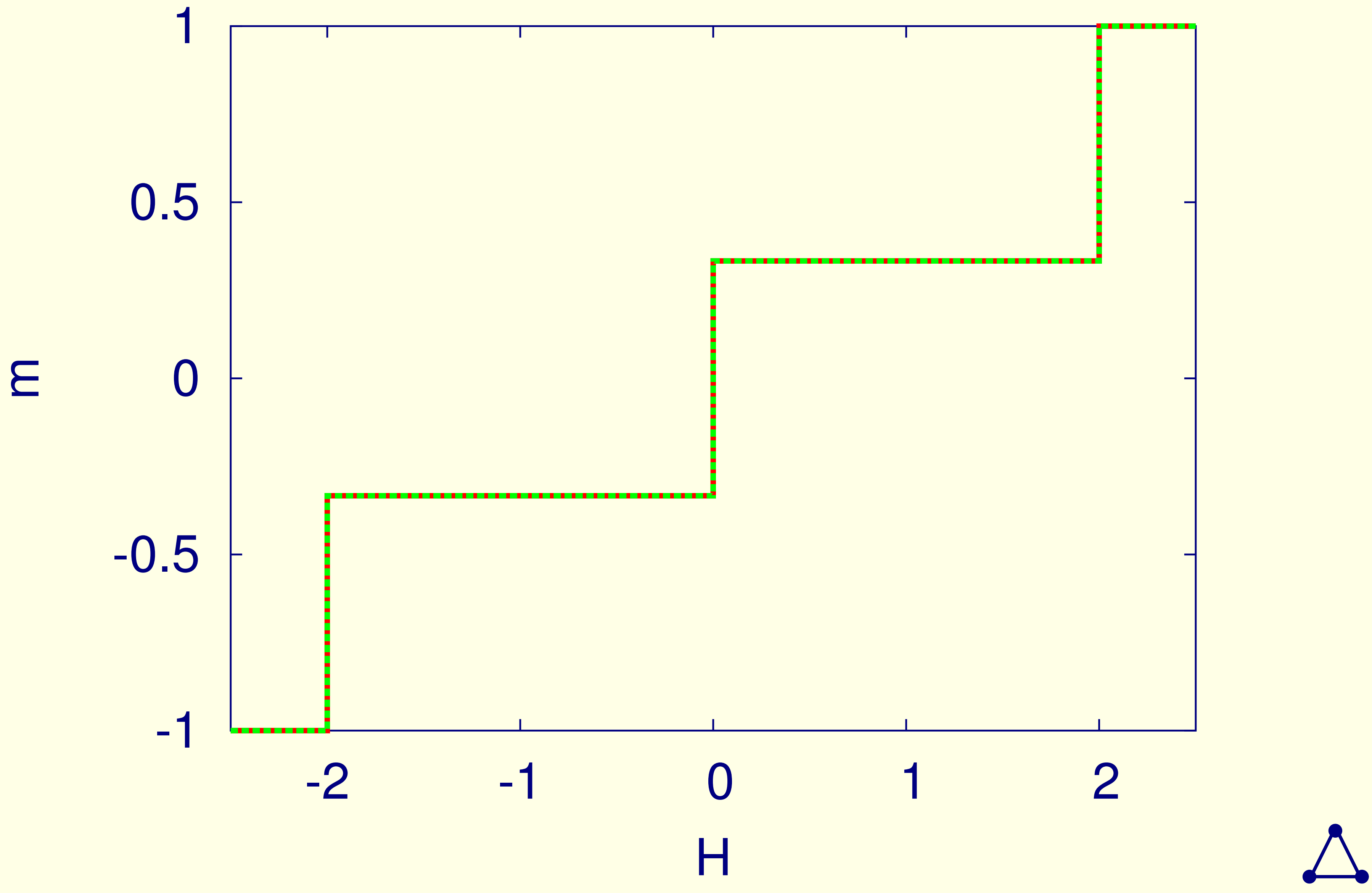


m
1
0.5
0
-0.5
-1
-2
-1
0
1
2
H

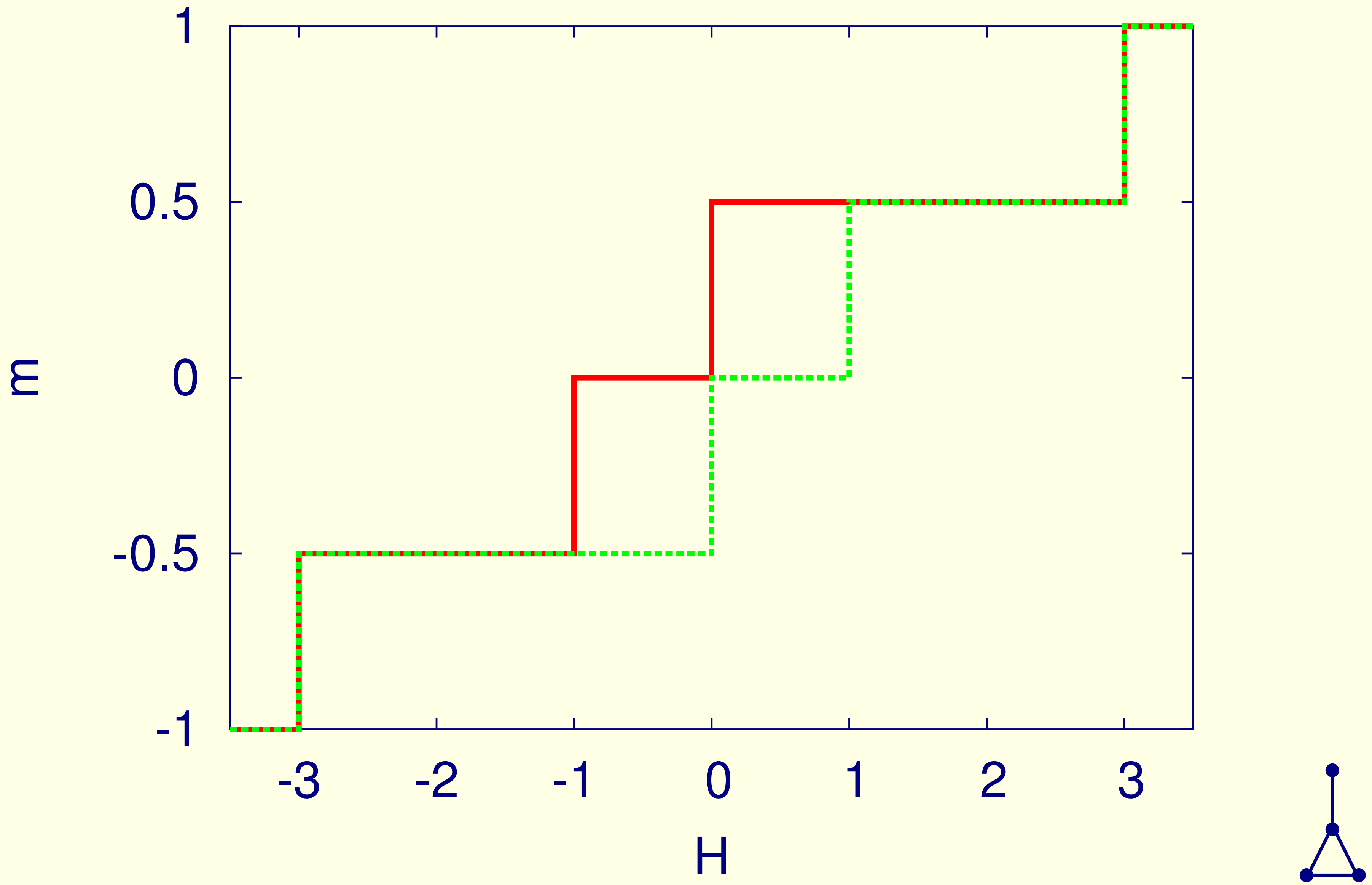


1
0.5
0
-0.5
-1
m
-2
-1
0
1
2
H

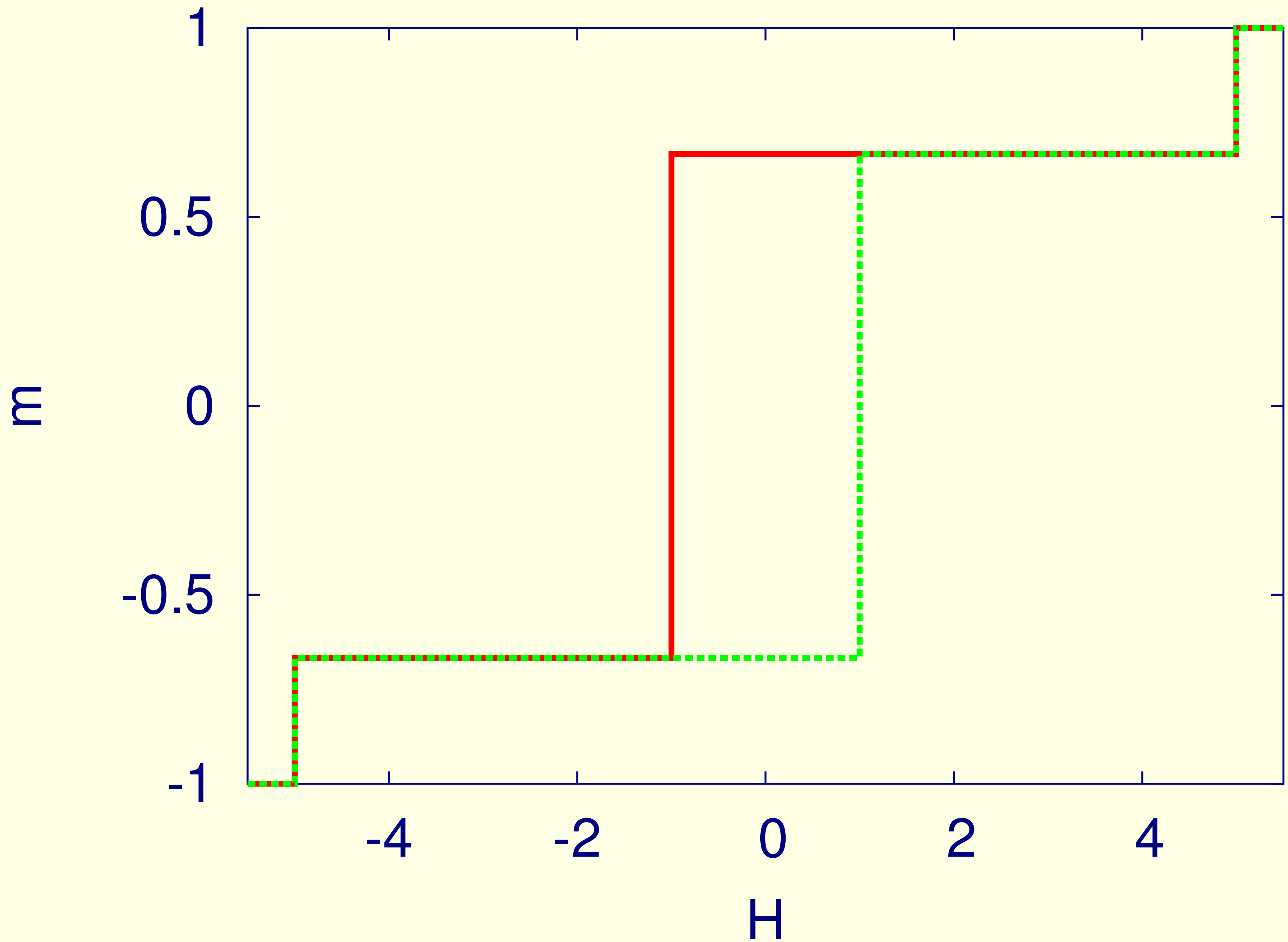




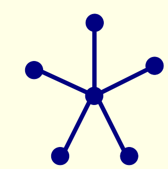
1
0.5
0
-0.5
-1
m
-2
-1
0
1
2
H

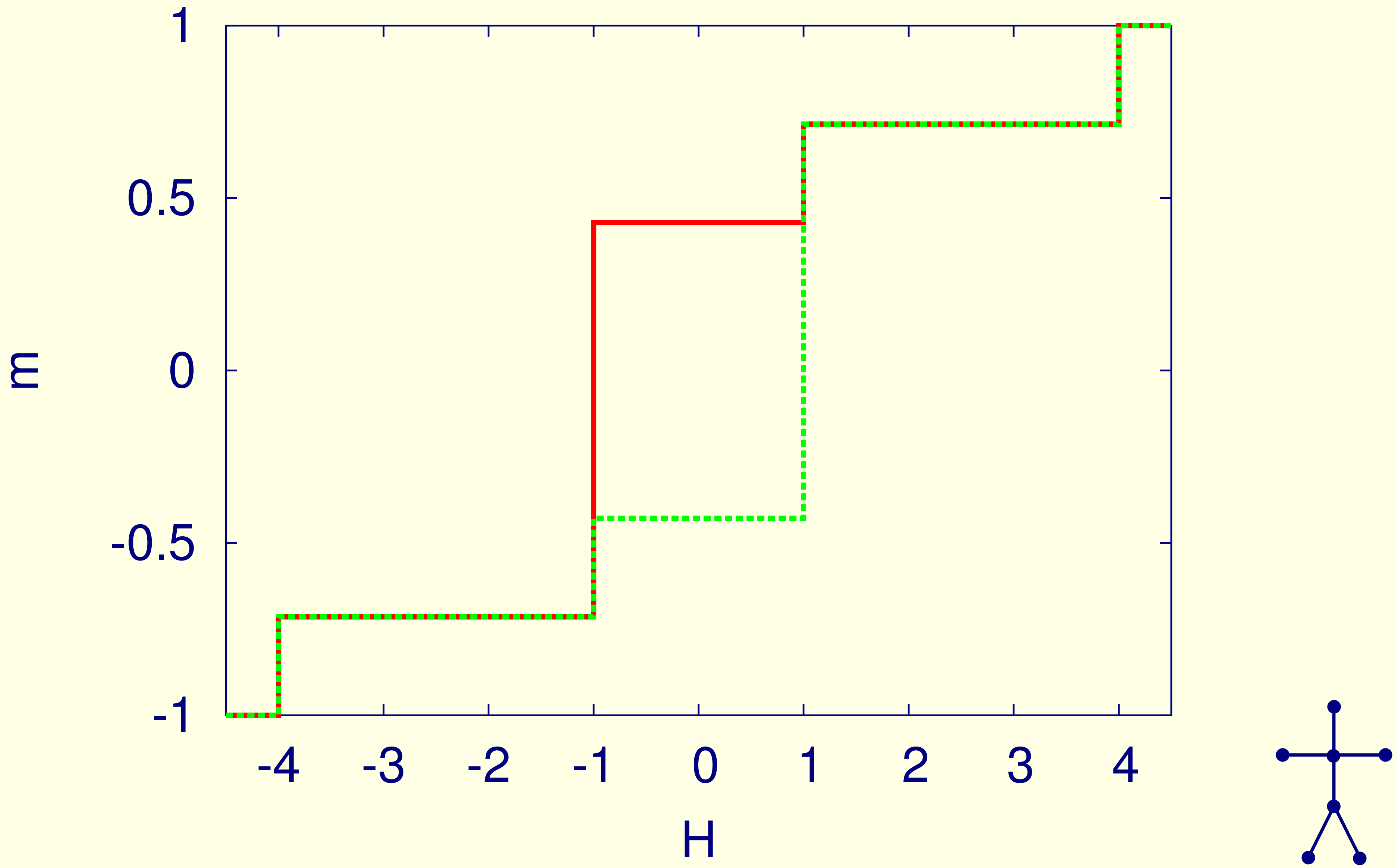


1
0.5
0
-0.5
-1
m
-3
-2
-1
0
1
2
3
H



1
0.5
0
-0.5
-1
m
-4
-2
0
2
4
H

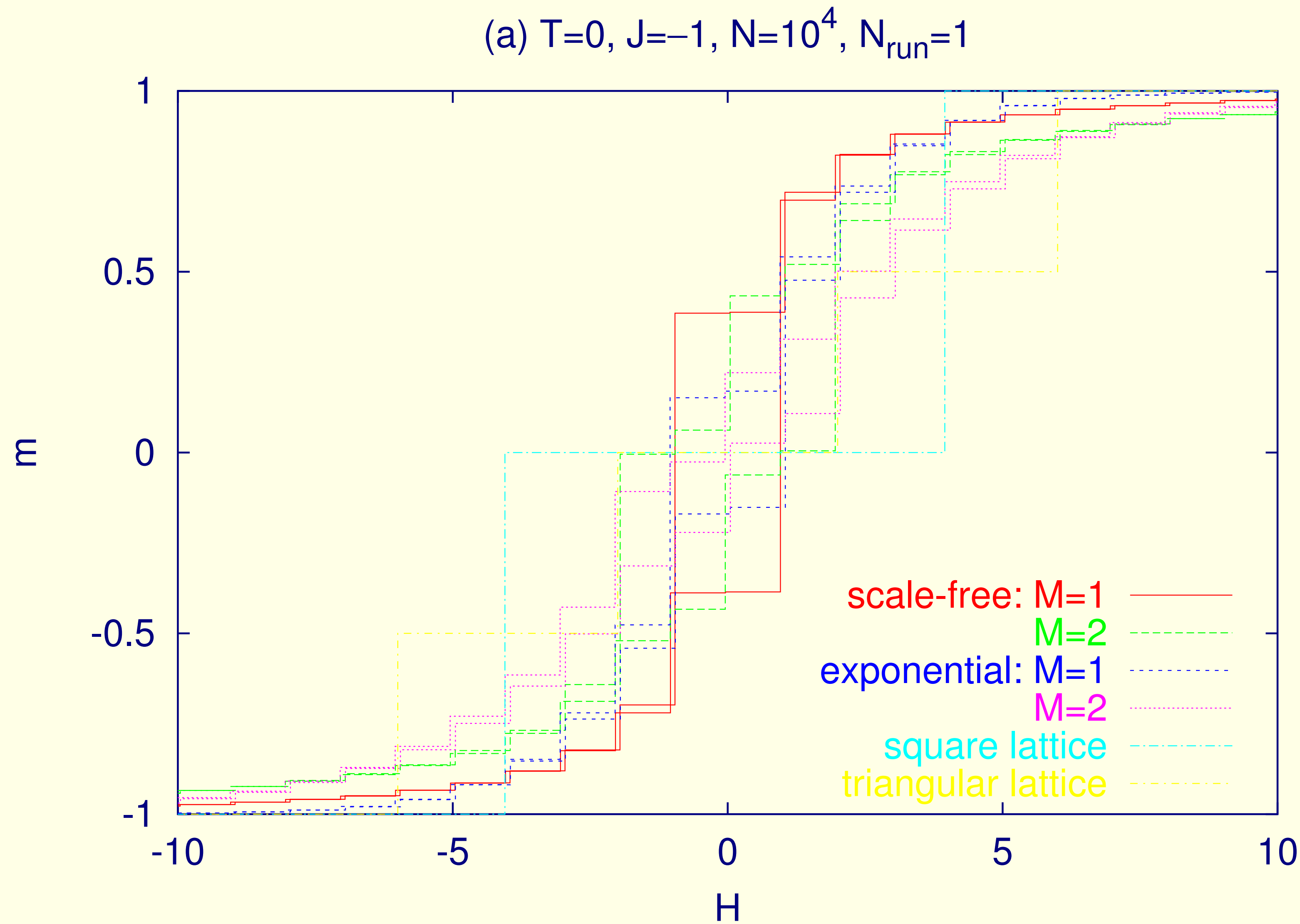




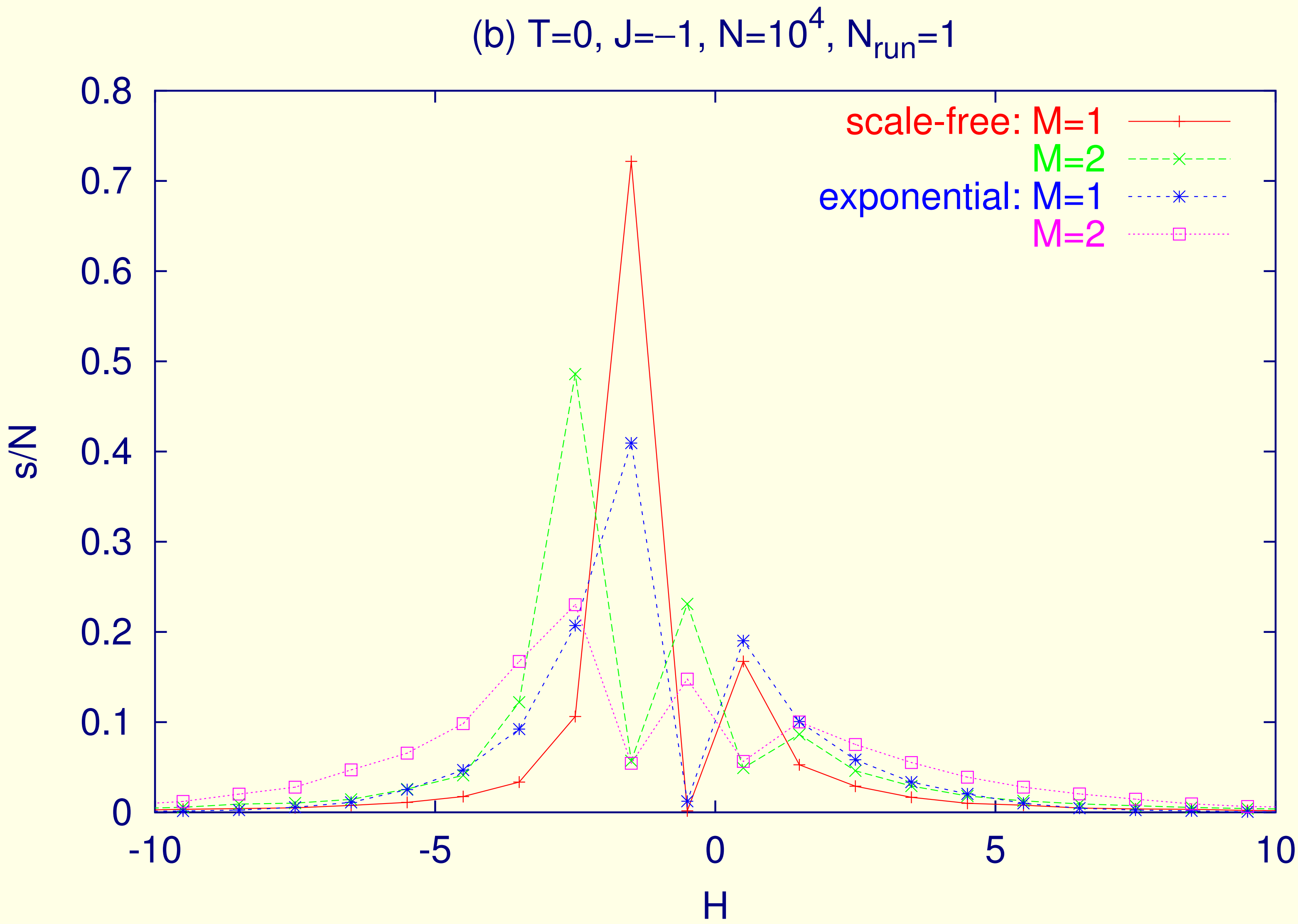
1
0.5
0
-0.5
-1
m
-4
-3
-2
-1
0
1
2
3
4
H

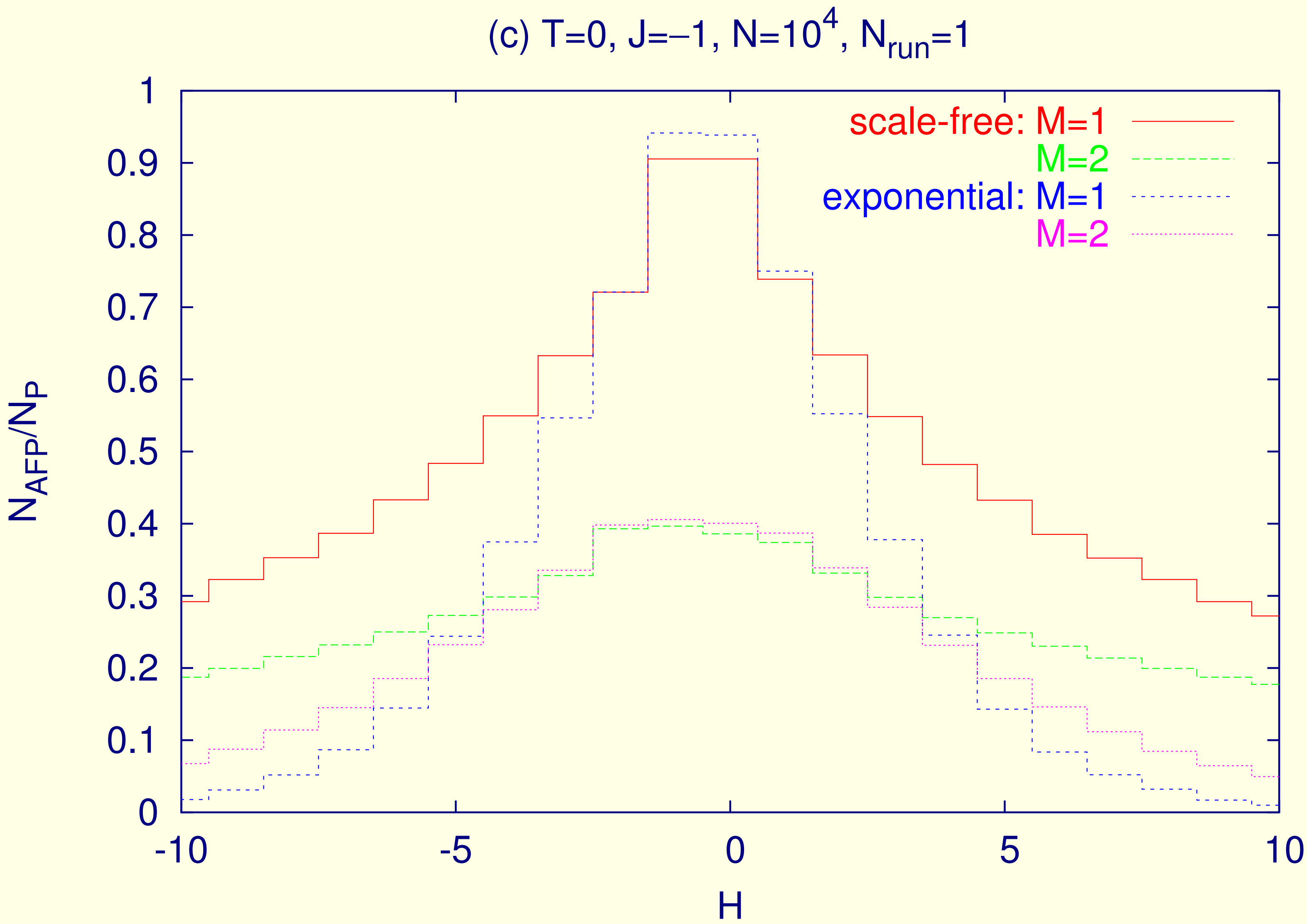
2.3 H

The field dependence of magnetization $m(H)$ total number of flipped spins $s(H)/N$ in given field H and density of walls (the number of antiferromagnetic pairs $N_{AFT}(H)$) normalized to the total number of bonds N_P for various kind of networks.



(a) T=0, J=−1, N=10^4, N_run=1
m
H
1
0.5
0
-0.5
-1
-10
-5
0
5
10
scale-free: M=1
M=2
exponential: M=1
M=2
square lattice
triangular lattice


(b) T=0, J=–1, N=10^4, N_run=1
scale-free: M=1
M=2
exponential: M=1
M=2
s/N
H
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-5
0
5
10

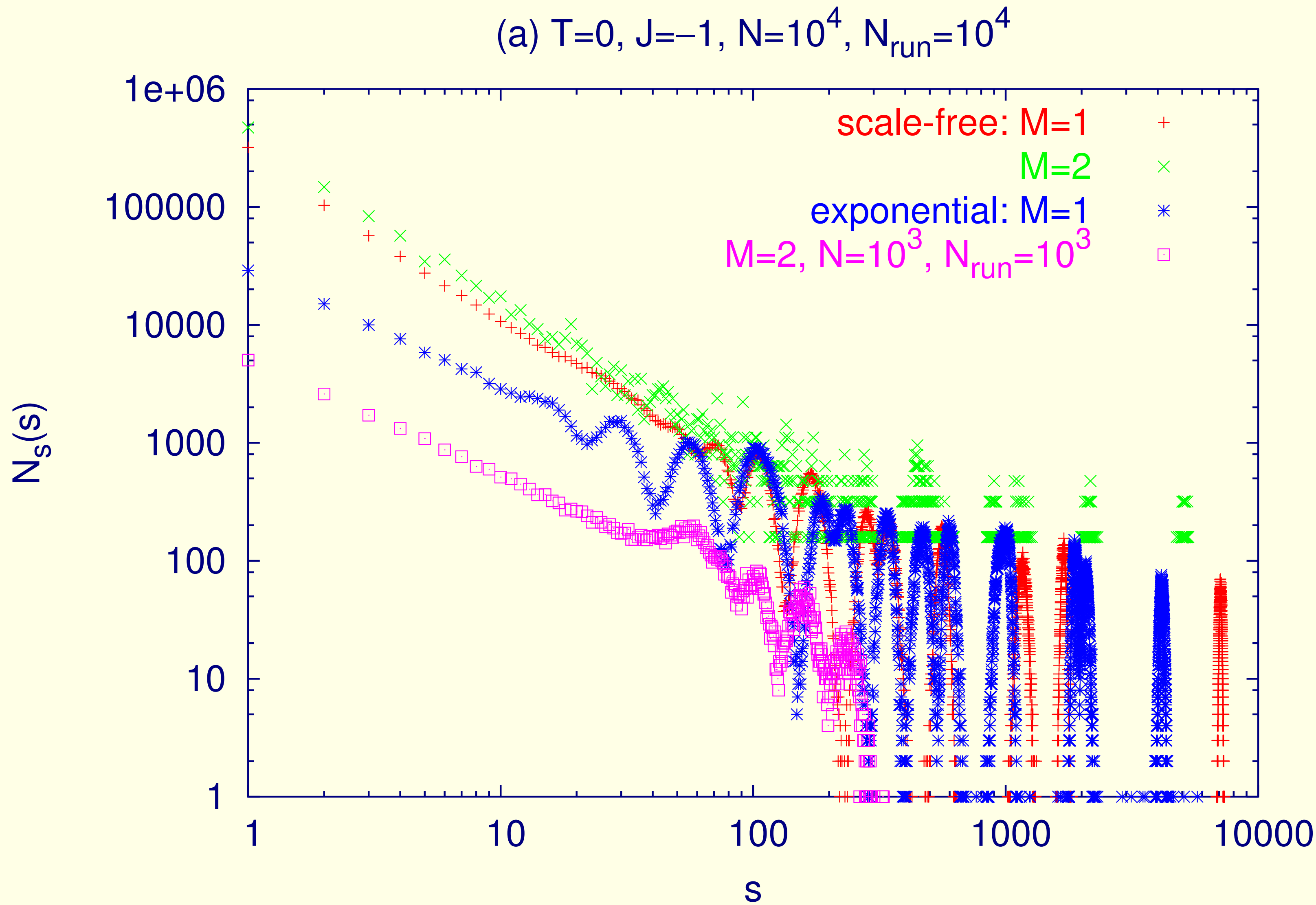


(c) T=0, J=−1, N=10^4, N_run=1
scale-free: M=1
M=2
exponential: M=1
M=2
N_AFP/N_P
H
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-5
0
5
10

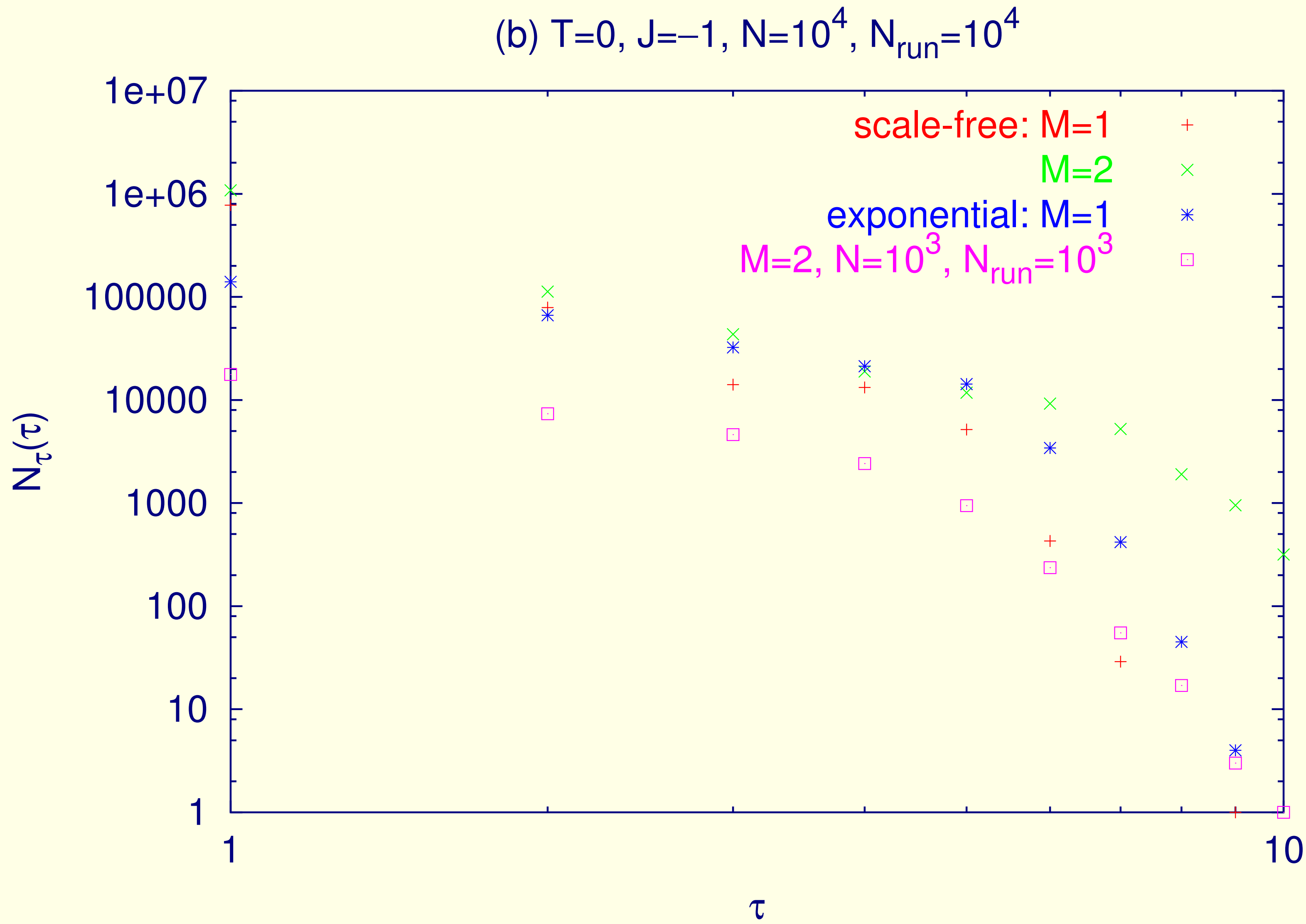
2.4 Histograms

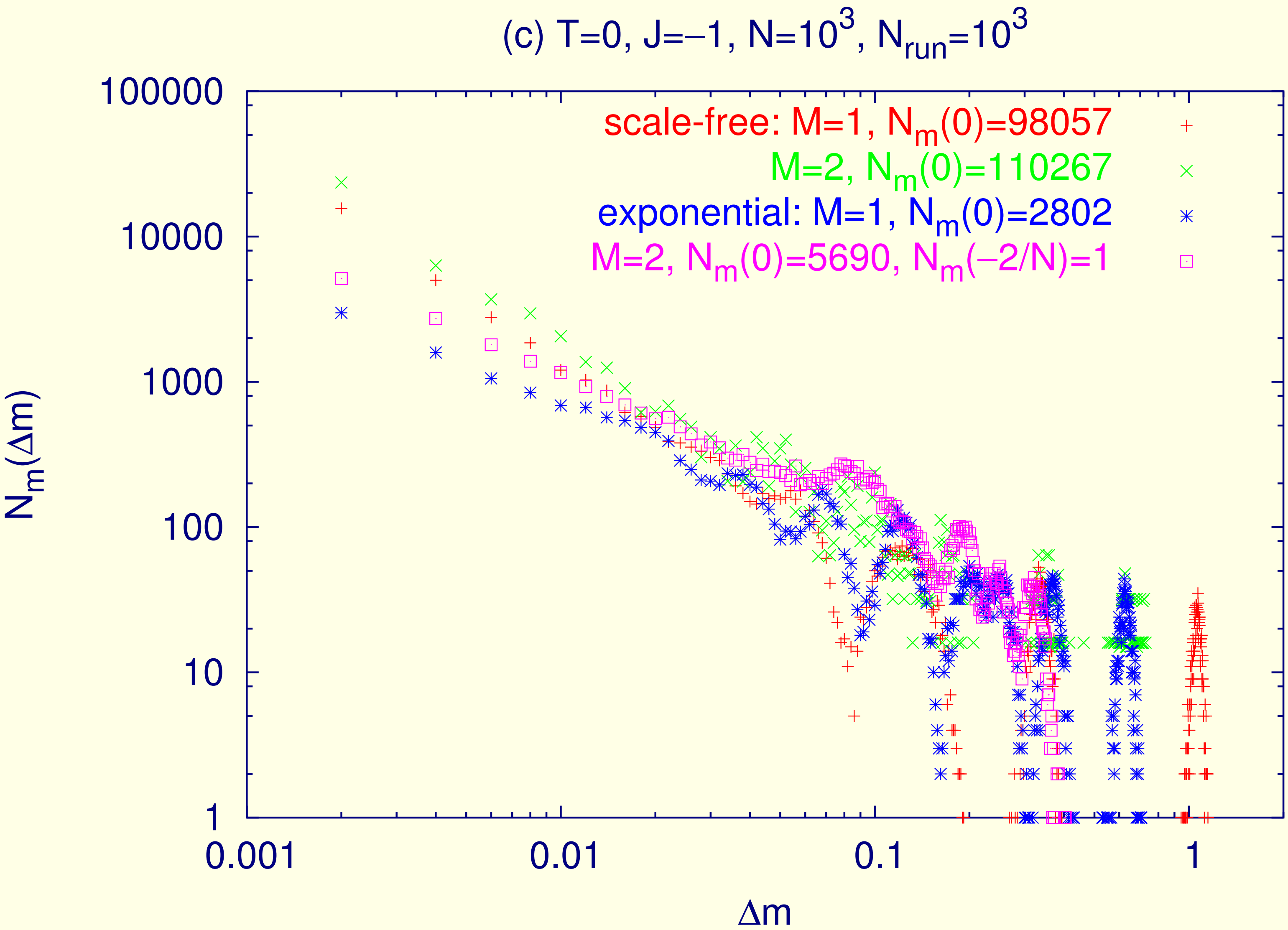
Histograms of size s, duration time τ, and changes of magnetization Δ_m of avalanches during hysteresis experiment.

Basing on our numerical results, we expect that true criticality can be proved i.e.: $N_s(s) \propto s^{-\alpha}$ and $N_m(\Delta_m) \propto \Delta_m^{-\beta}$ for $M = 1, 2$.



(a) T=0, J=−1, N=10^4, N_run=10^4
1e+06
100000
10000
1000
100
10
1
N_s(s)
1
10
100
1000
10000
s
scale-free: M=1
M=2
exponential: M=1
M=2, N=10^3, N_run=10^3


(b) T=0, J=−1, N=10^4, N_run=10^4
1e+07
1e+06
100000
10000
1000
100
10
1
1
10
N_τ(τ)
τ
scale-free: M=1
M=2
exponential: M=1
M=2, N=10^3, N_run=10^3



(c) T=0, J=−1, N=10^3, N_run=10^3
scale-free: M=1, N_m(0)=98057
M=2, N_m(0)=110267
exponential: M=1, N_m(0)=2802
M=2, N_m(0)=5690, N_m(−2/N)=1
N_m(Δm)
Δm
100000
10000
1000
100
10
1
0.001
0.01
0.1
1

2.5 Size Evaluation of the Larges Avalanches

Fraction of spins flipped at $H = -M$ and maximal avalanche size s_{max}/N as compared with fraction of spins with degree equal to M for various networks.

Degree distributions for exponantial

$$P_k(k) = \begin{cases} 2^{-k} & \text{for } M = 1, \\ 3/4 \cdot (3/2)^{-k} & \text{for } M = 2, \end{cases}$$

and scale-free networks

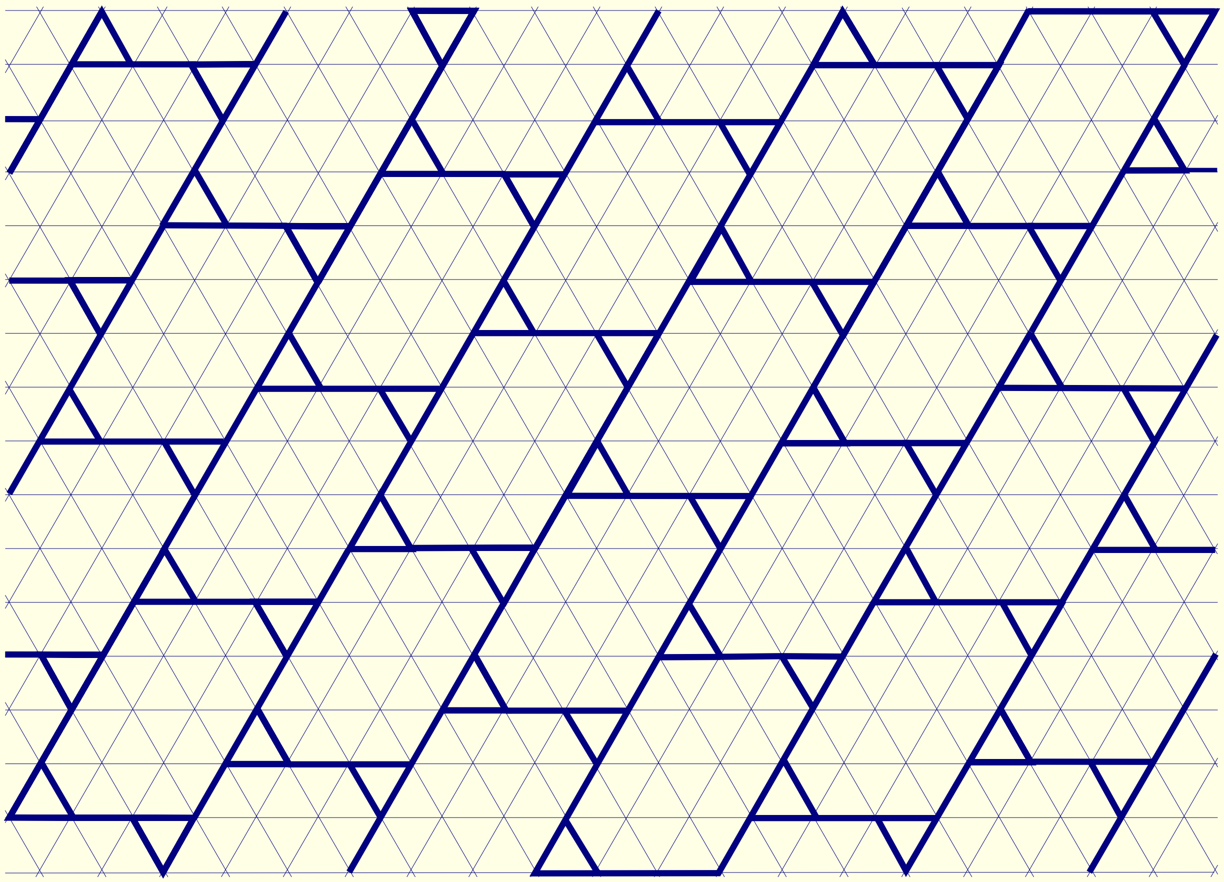
$$P_k(k) = \frac{2M(M+1)}{k(k+1)(k+2)}.$$

	exponential	
M	1	2
s for $H = -M$	0.4094	0.2302
s_{max}/N	0.42	0.24
$P_k(k = M)$	1/2	1/3
α	1.00	0.99
β	0.92	0.93
ℓ	15.58	4.9^*

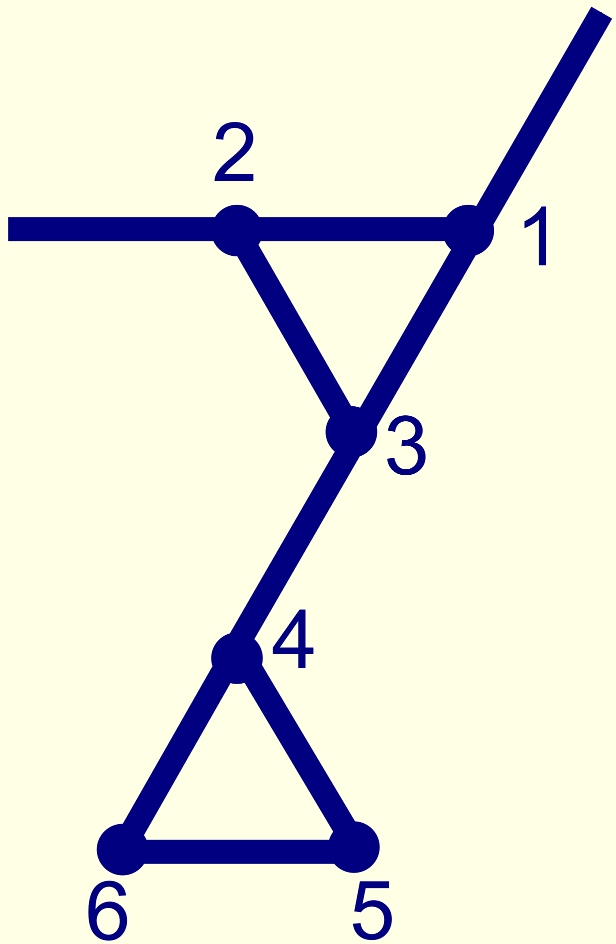
	scale-free	
M	1	2
s for $H = -M$	0.7217	0.4858
s_{max}/N	0.71	0.51
$P_k(k = M)$	2/3	1/2
α	1.46	1.44
β	1.56	1.48
ℓ	9.1	5.1

3 Complex Lattices

3.1 Archimedean $(3, 12^2)$ Lattice Structure

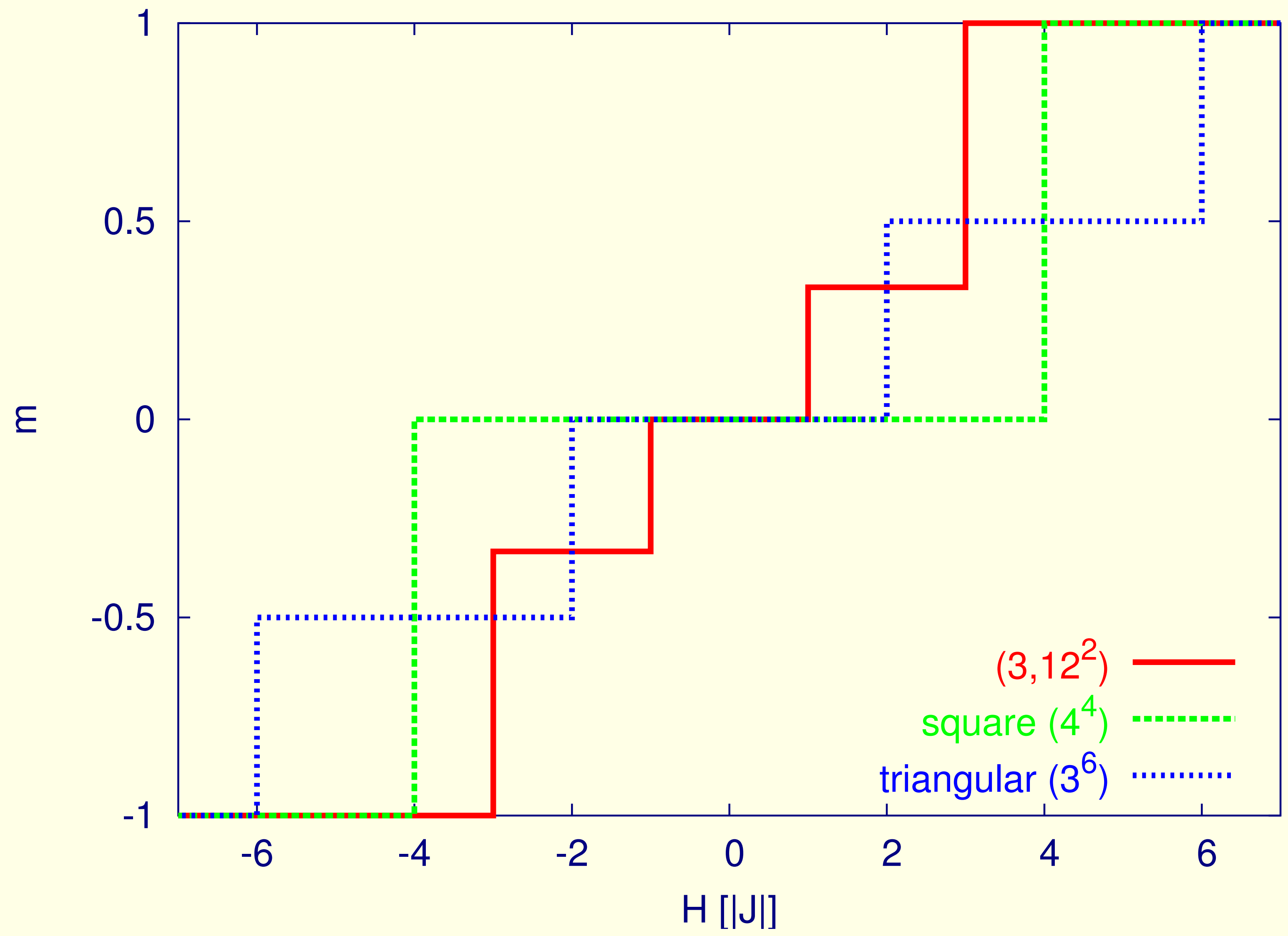


A unit cell:



3.2 Magnetic Properties

The magnetization curves for $(3, 12^2)$, the square lattice and the triangular lattice (solid, dashed and dotted lines, respectively) for $T = 0$.



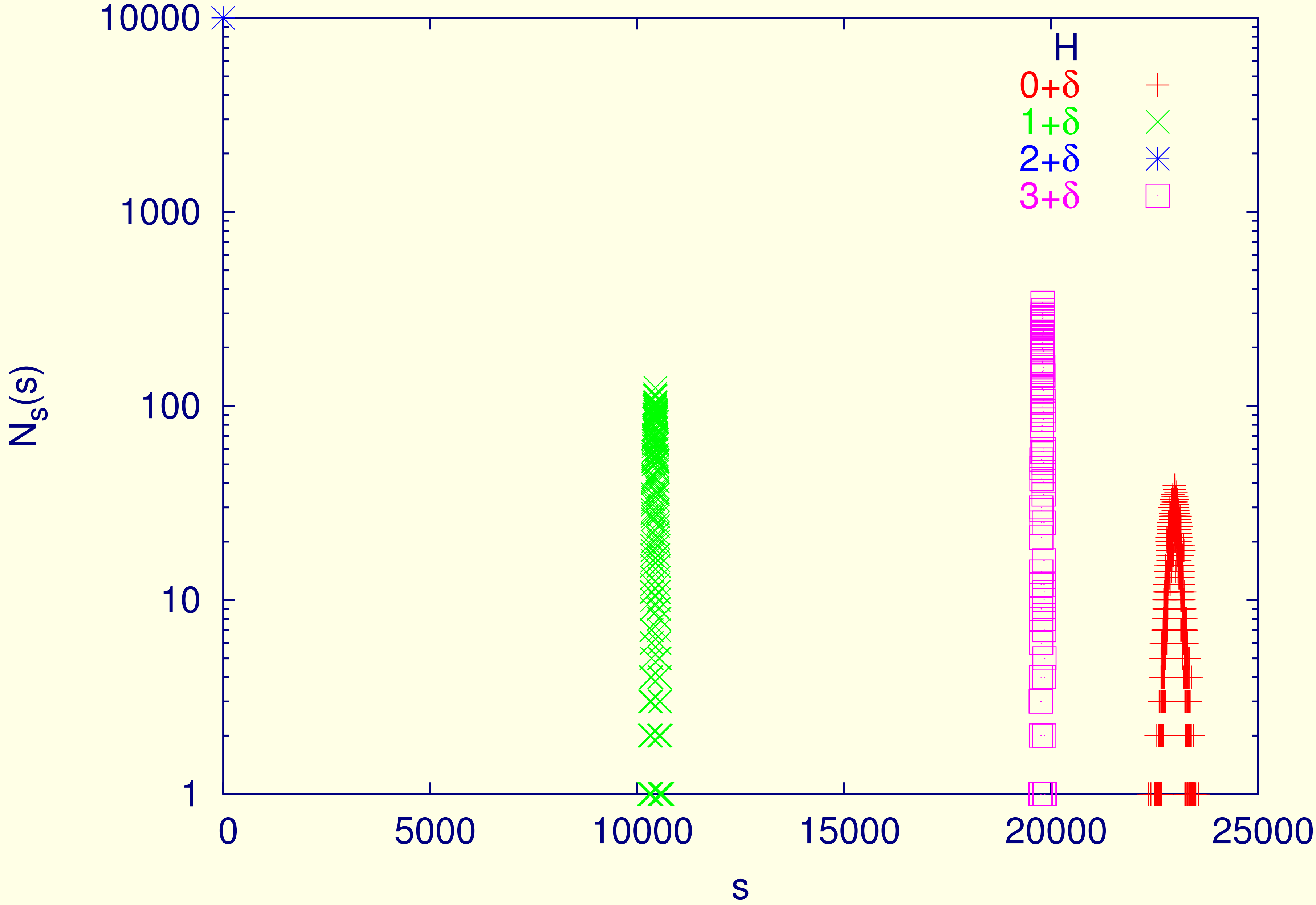
1
0.5
0
-0.5
-1
m
-6
-4
-2
0
2
4
6
H [|J|]
(3,12^2)
square (4^4)
triangular (3^6)

3.3 The Spectrum of Avalanches

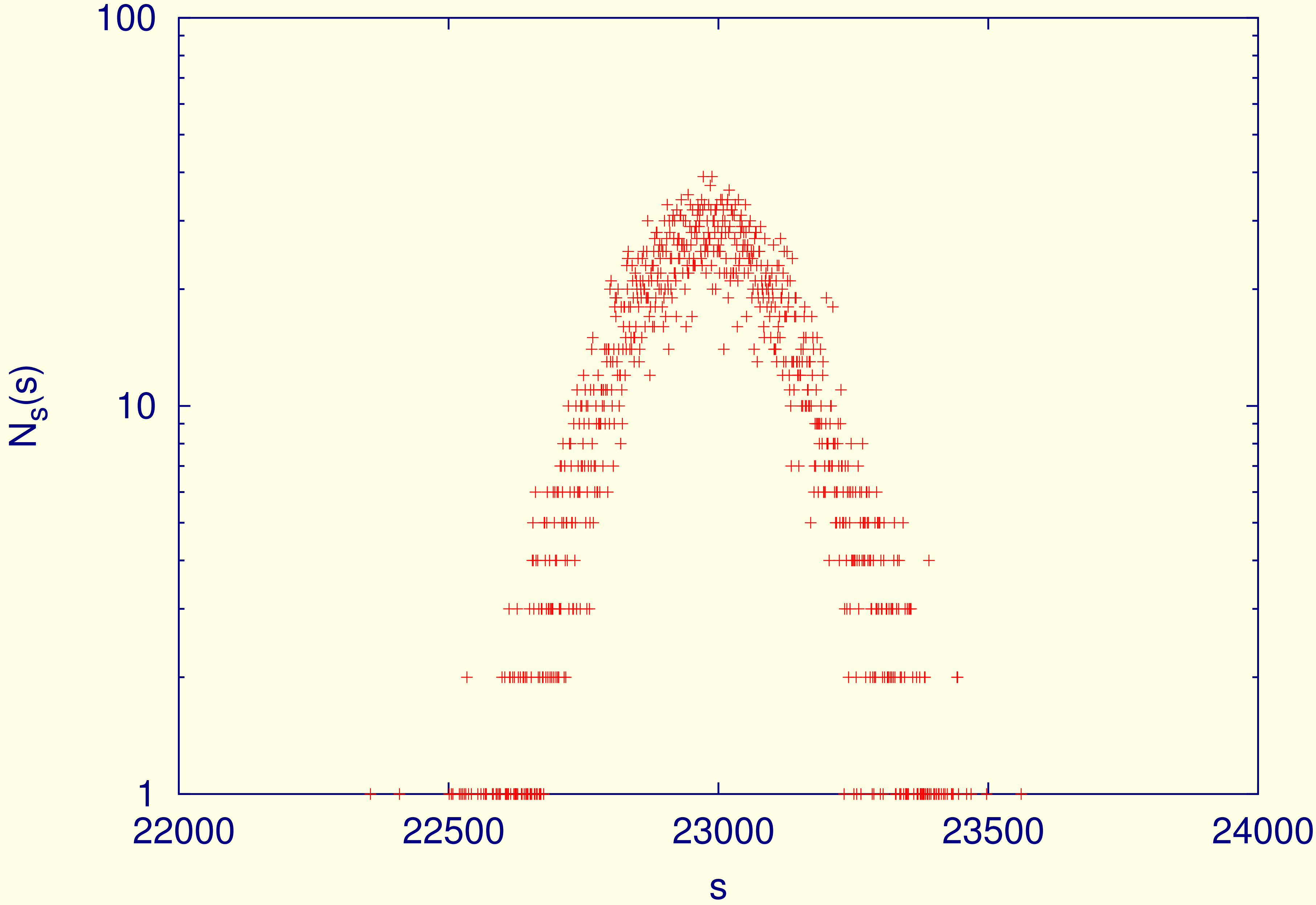
Three peaks are obtained for avalanches at three different fields: $H = 0$, $H = 1\ [|J|]$, $H = 3\ [|J|]$.

This result reflects the distribution of the interaction field.

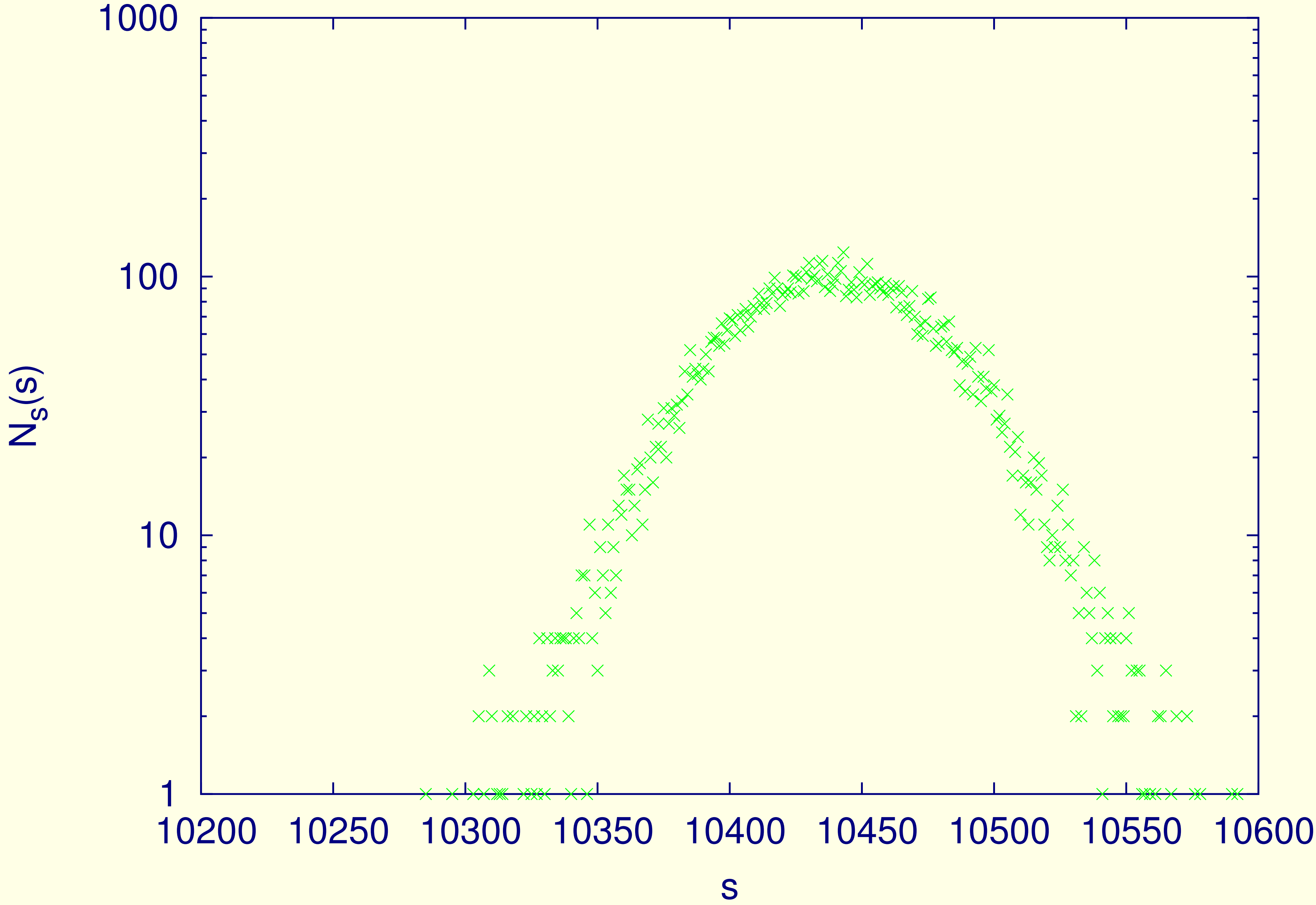
$T = 0$, $N = 6 \cdot 10^4$, $N_{\text{run}} = 10^4$.

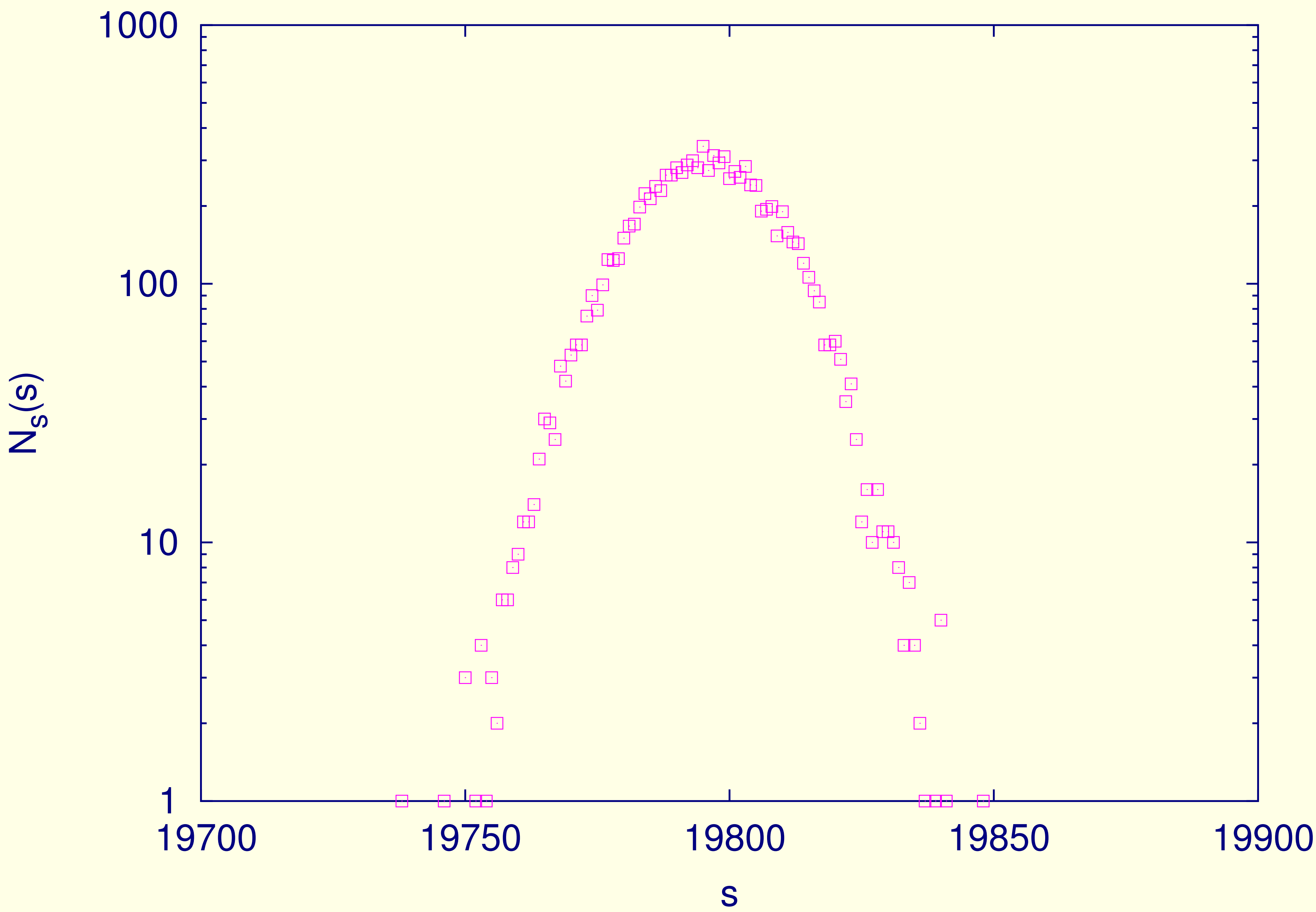


10000
1000
100
10
1
0
5000
10000
15000
20000
25000
s
N_s(s)
H
0+δ
1+δ
2+δ
3+δ



100
10
1
22000
22500
23000
23500
24000
$N_s(s)$
s

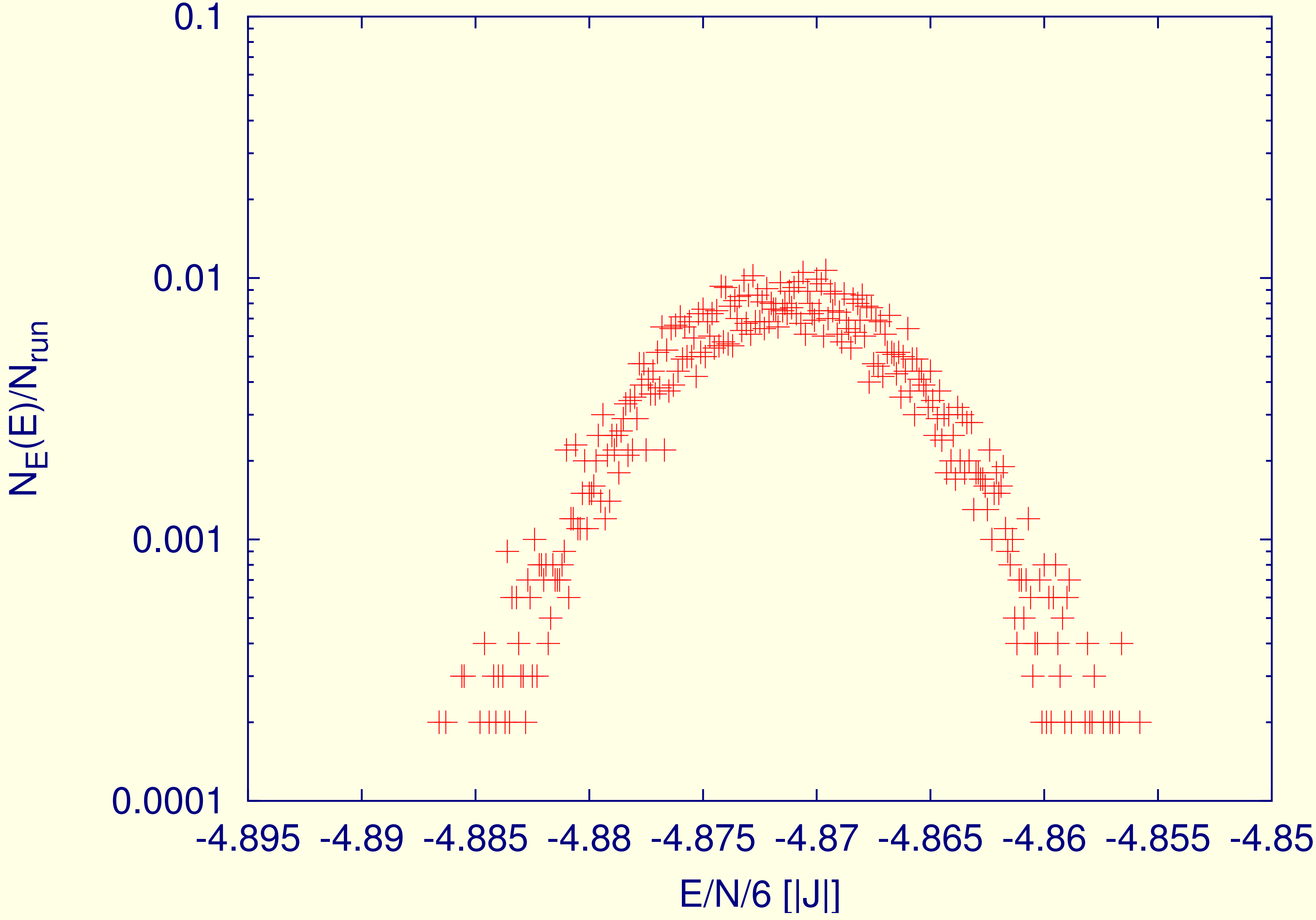

1000
100
10
1
$N_s(s)$
10200
10250
10300
10350
10400
10450
10500
10550
10600
s


1000
100
10
1
19700
19750
19800
19850
19900
s
$N_s(s)$

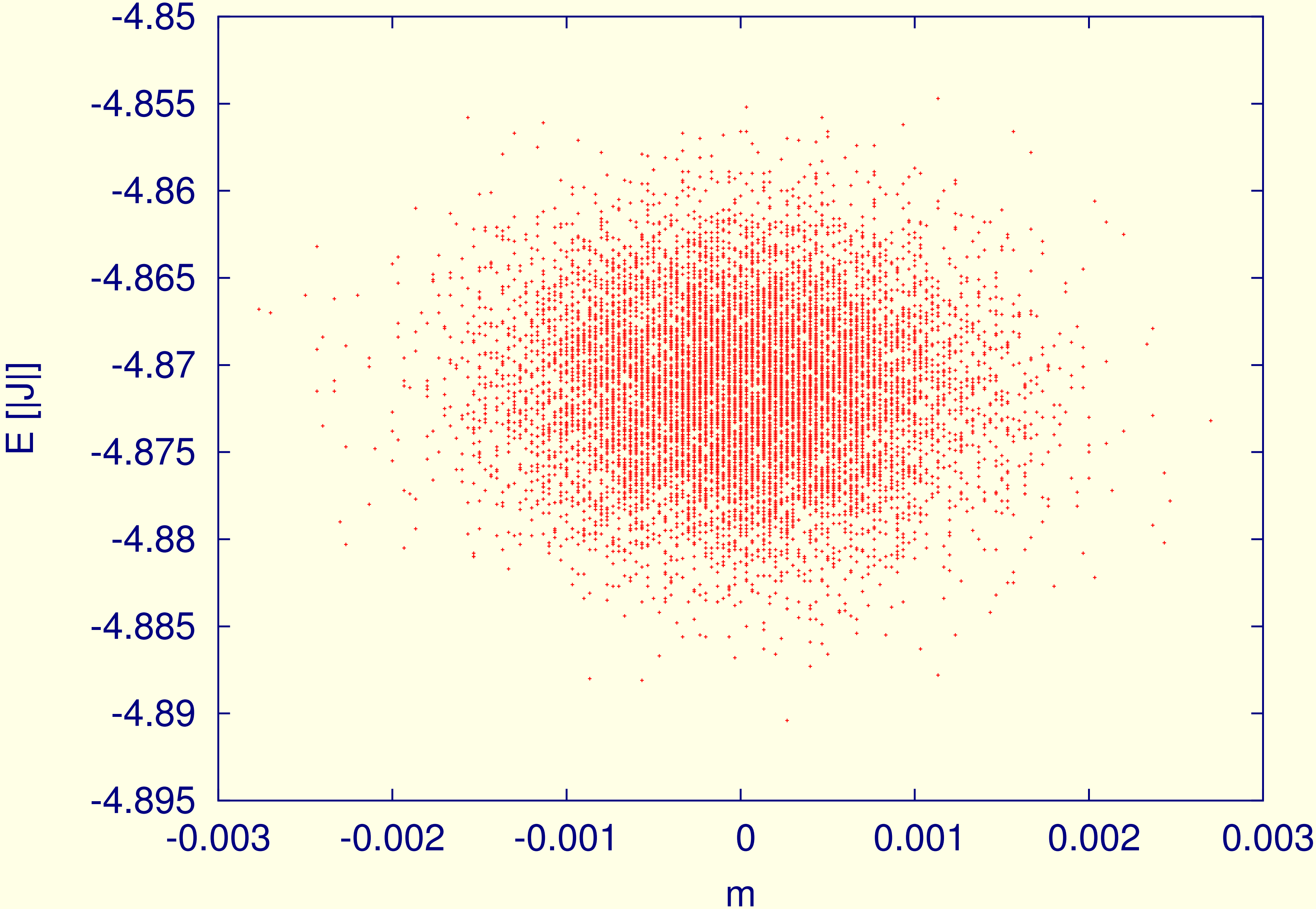
3.4 Energy Distribution

Energy distribution of metastable states obtained for $T = 0$ from random initial states. The mean energy is about 2.5 % higher that GSE=$5J$ per unit cell.

Energy against the reduced magnetization $m = M/N$ of the same metastable states. No visible correlation is found. $m(t = 0) = 0$, $H = 0$, $T = 0$, $N = 6 \cdot 10^4$, $N_{\text{run}} = 10^4$.



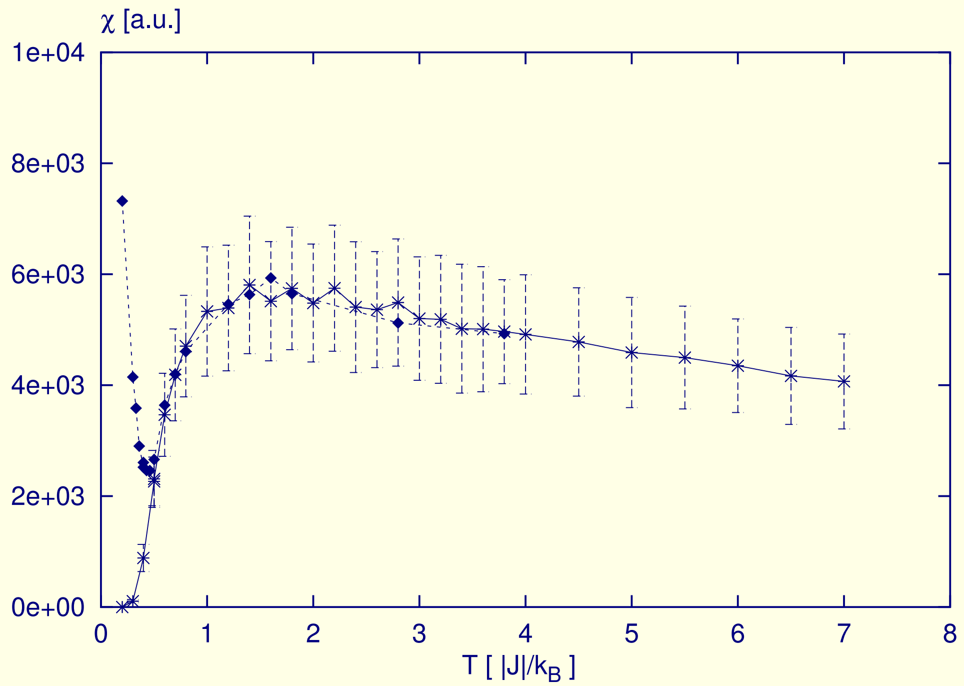
0.1
0.01
0.001
0.0001
$N_E(E)/N_{run}$
-4.895
-4.89
-4.885
-4.88
-4.875
-4.87
-4.865
-4.86
-4.855
-4.85
E/N/6 [|J|]



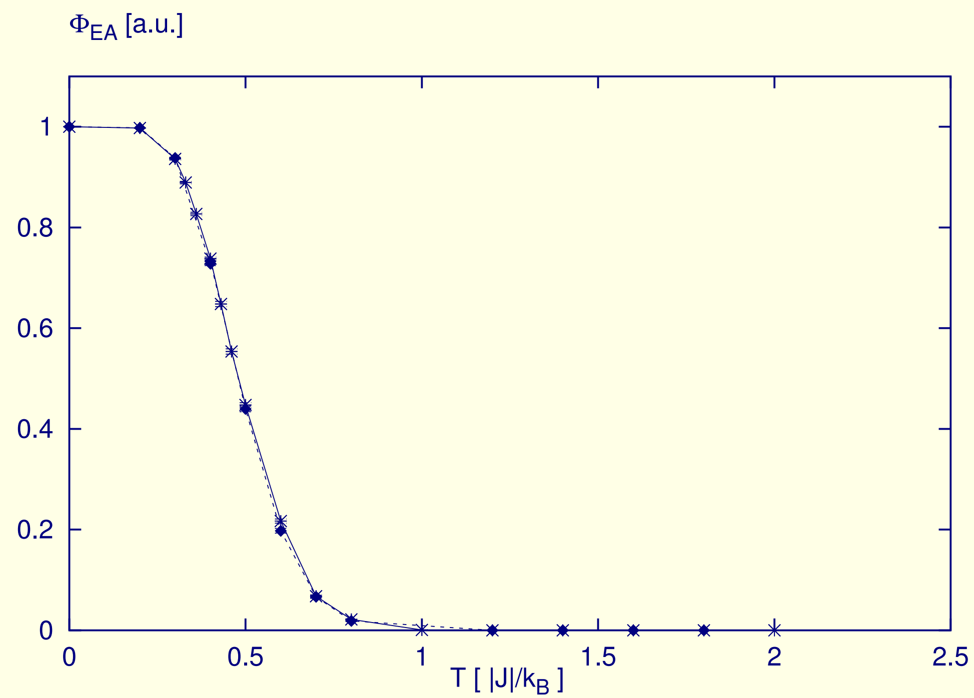
-4.85
-4.855
-4.86
-4.865
-4.87
-4.875
-4.88
-4.885
-4.89
-4.895
E [|J|]
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
m

3.5 Spin glass properties

The results for the saturated initial state (solid line) show that below the maximum, χ tends mildly to zero, with numerical uncertainties decreasing at low T.s The results for the random initial state (dotted curve) show irregularities below $T = 0.7\ [|J|/k_B]$. In this range of temperature, we observe a rapid increase of χ below the main maximum, and numerical uncertainties increase when T is lowered. For $T = 0.2\ [|J|/k_B]$, the uncertainties are the largest and reach about 100 percent of the obtained value.



Thermal dependence of the Edwards–Anderson order parameter. The results are practically the same for initial random state and initial saturated state.



4 Conclusions

4.1 Spin avalanches

We discuss magnetic properties of growing exponential and scale-free trees and graphs, decorated with spins, with antiferromagnetic interaction.

The hysteresis loops are obtained by numerical experiments, most of them performed for $T = 0$.

The loops are found to contain small and large avalanches, both for number of flipping spins and for the changes of magnetization.

The statistics of small avalanches is ruled by power laws.

Size of largest avalanches are approximately equal to the number $NP_k(k = k_{\mathrm{min}})$ of nodes with smallest possible degree.

For descending field, most of spins at these nodes flip at the applied field $H = -k_{\text{min}}$.

4.2 Spin glasses

Numerical results reported above suggest, that the temperature of the transition between the paramagnetic state and the low-temperature state, which seems to be the spin-glass state, is positive.

It is an open question, how much disorder is needed to reproduce the vanishing of the transition temperature, which is the standard result for two-dimensional Ising spin glasses [**?**].

With small amount of disorder, we expect that the interaction field at some sites is zero and these spins flip freely.

Once a cluster of these spins spans throughout the lattice, the transition at finite temperatures is likely to disappear.

Acknowledgements

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References

[1] B.Tadić, K.Malarz, K.Kułakowski, `cond-mat/0411223`.

[2] K.Malarz, J.Karpińska, K.Kułakowski, B.Tadić, — in preparation.

[3] M.J.Krawczyk, K.Malarz, B.Kawecka-Magiera, A.Z.Maksymowicz, K.Kułakowski, `cond-mat/0412545`.