Square lattice site percolation at increasing ranges of neighbor bonds

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1 Introduction

- Percolation = geometrical model of a first order phase transition
- Percolation threshold p_c
- Bond and site percolation

- Universal critical exponents

$$P_\infty \propto (p - p_c)^\beta$$

$$\xi \propto |p - p_c|^{-\nu}$$

$$S \propto |p - p_c|^{-\gamma}$$

- T, T_c, m, ξ, χ

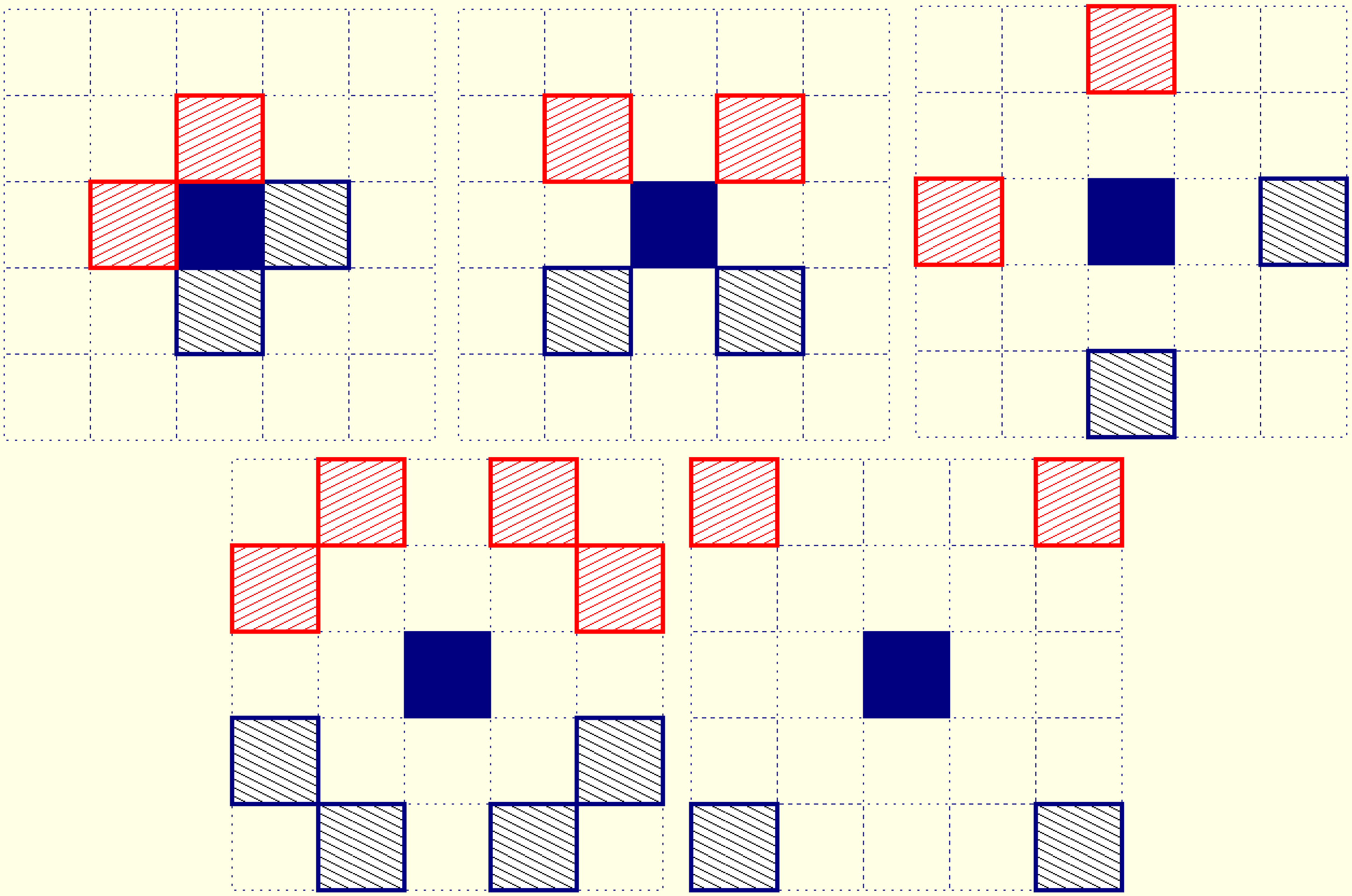
1.1 Percolation thresholds

- thresholds up to $d = 13$ for the hypercube
- complex (scale-free for example) networks
- regular lattices with neighbor links which are N^2 = von Neumann's neighborhood and/or (N^2+N^3) = Moore's neighborhood

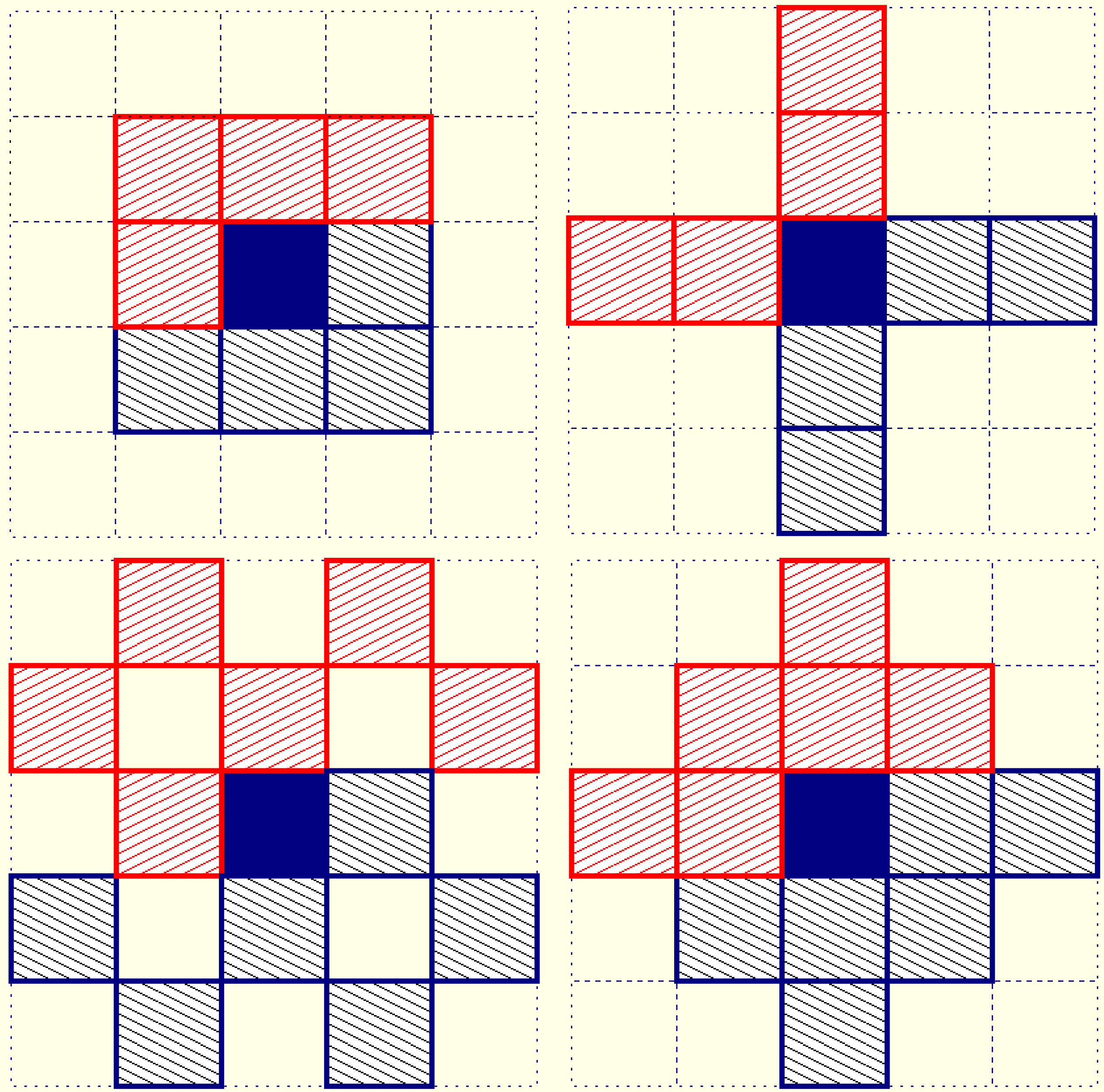
We consider the series of

- nearest neighbors (N^2),
- next-nearest neighbors (N^3),
- next-next-nearest neighbors (N^4),
- 4th-nearest neighbors (N^5),
- and 5th-nearest neighbors (N^6).

for each one of the considered distance of bonds, all others are not active.



We then consider combinations of various ranges of neighborhoods with (N^2+N^3), (N^2+N^4), (N^2+N^5) and $(N^2+N^3+N^4)$.



1.2 Restoring sites percolation

- How to restore site percolation on a square lattice with nearest neighbor links (N^2) once a random attack has occurred?

- We consider the square lattice at the percolation threshold $p_c \equiv p_c(\mathrm{N}^2)$ once a fraction x of the initial p_c sites have been randomly destroyed.

- Two strategies are suggested.

1. A density y of new sites are created on the empty sites with longer range bonds, either next-nearest neighbor (N^3) or next-next-nearest neighbor (N^4). It is worth to stress that these additional sites with N^3 or N^4 links have no N^2 links.

2. No additional sites are created but instead new longer range links, either N^3 or N^4, are added to the N^2 links but only for a fraction v of the remaining not damaged $(p_c - x)$ sites. Accordingly $v(p_c - x)$ sites have N^2 plus either N^3 or N^4 links while $(1-v)(p_c - x)$ sites have only their initial N^2 links.

Given a fixed density of destroyed sites x, the associated values y_c and v_c which restore site percolation are calculated with respectively N^3 and N^4. Results are obtained for the whole range $0 \leq x \leq p_c$, which in turn leads to new site percolation thresholds, $\pi_3 \equiv (p_c - x + y_c)$ for N^3, $\pi_4 \equiv (p_c - x + y_c)$ for N^4, $\pi_{23} \equiv v_c(p_c - x)$ for N^3 and $\pi_{24} \equiv v_c(p_c - x)$ for N^4.

Since for each strategy we have two kinds of sites with respect to their links, we consider instead of above problem a square lattice with a density π of occupied sites, of which a given fraction q has one kind of links, the initial N^2, while remaining fraction $(1-q)$ have the other kind, N^3, N^4, (N^2+N^3) or (N^2+N^4).

Then, given the mixing neighborhood parameter q, we evaluate the threshold π_c using Monte Carlo simulations. From π_c we can then go back to our former problem and extract the values of the pairs $\{x, y_c\}$ and $\{x, v_c\}$. For the first strategy $x = p_c - q\pi_c$ and $y_c = (1-q)\pi_c$ while for the second $x = p_c - \pi_c$ and $v_c = (1-q)$. Monte Carlo simulations are run over the whole range $0 \leq q \leq 1$.

2 Calculation

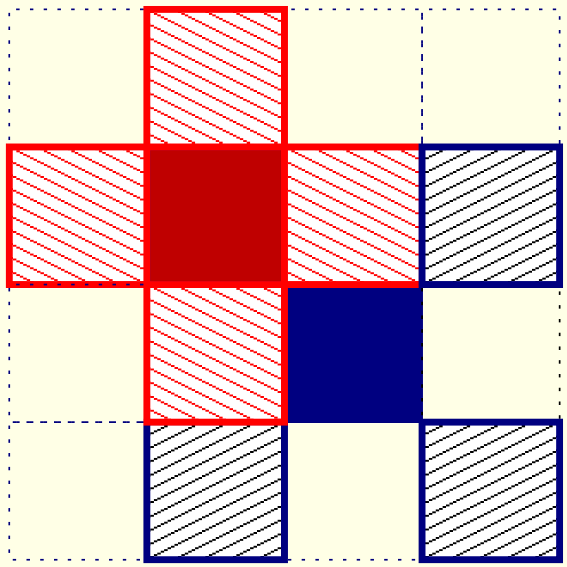
There exist several computational techniques which allow to perform calculations of percolation thresholds

- Newman + Ziff, PRL **85**, 4104 (2000)
- Ziff, J. Stat. Phys. **28**, 838 (1982)
- Leath, PRB **14**, 5046 (1976)
- Hoshen + Kopelman, PRB **14**, 3428 (1976)

2.1 HK algorithm

Once the lattice is given with the occupied sites, it allows to recognize which sites belong to which clusters. With HKA one can assign to each occupied site a label and sites in the same cluster have the same labels. Different labels are assigned to different clusters.

2.2 Complex neighbourhoods



Are full-filled black and dark (red) sites in one cluster? The dark (red) site has N^2 while the black one N^3. The answer is sites labeling order dependent.

2.3 Semi-empirical GM formula

p_c is known analytically for

$$p_c(TR - site) = 1/2$$

$$p_c(SQ - bond) = 1/2$$

$$p_c(TR - bond) = 2/\sin(\pi/18)$$

$$p_c(HC - bond) = 1 - 2/\sin(\pi/18)$$

$$p_c = p_0[(d-1)(z-1)]^{-a}d^b$$

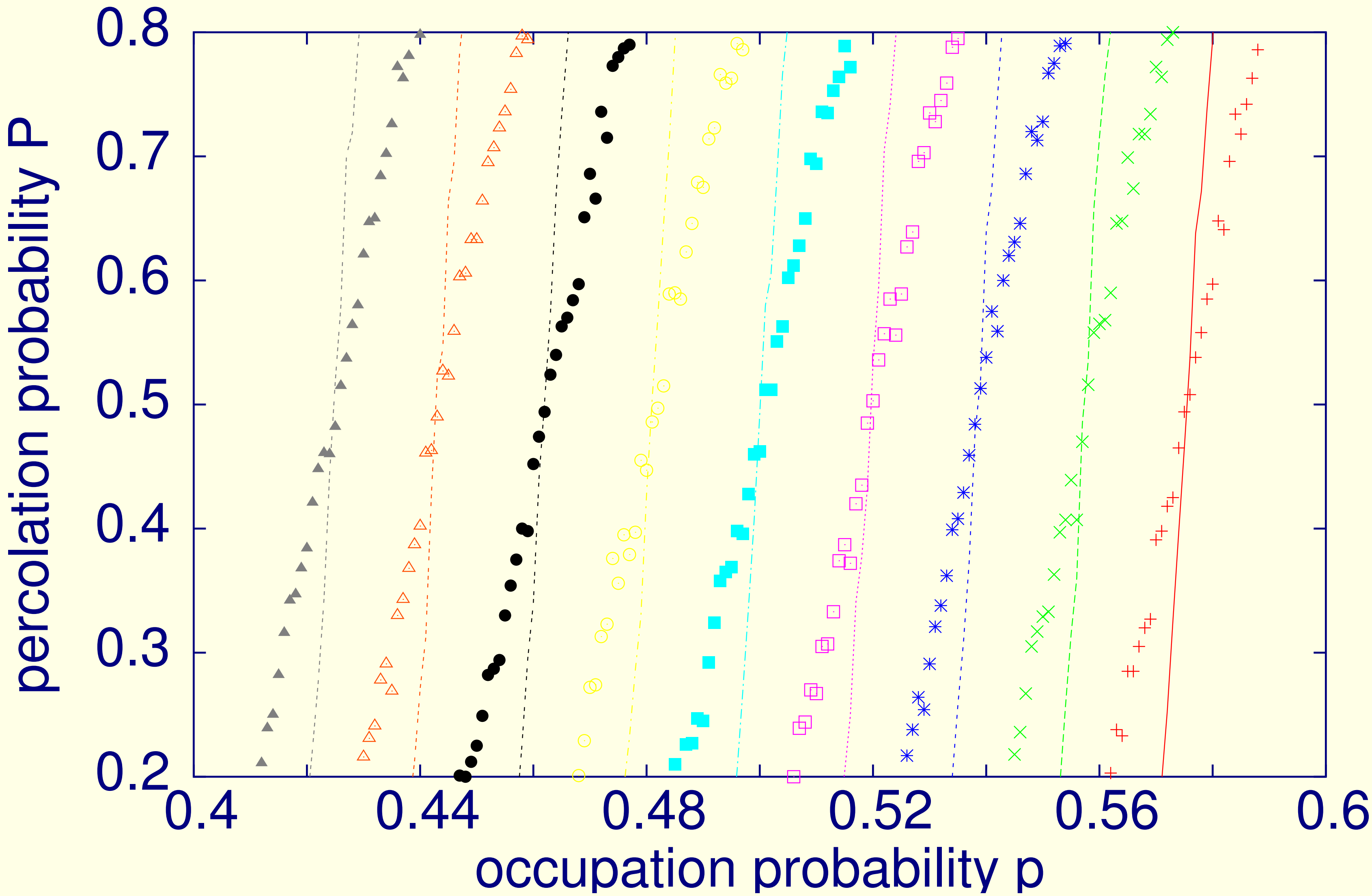
All thresholds up to $d \to \infty$ are found to belong to only three universality classes defined by a set of value for $\{p_0, a\}$.

$$p_c(NN+NNN) = 1 - p_c(NN)$$

$$p_c(NN) = 0.5927460\cdots$$

3 Results

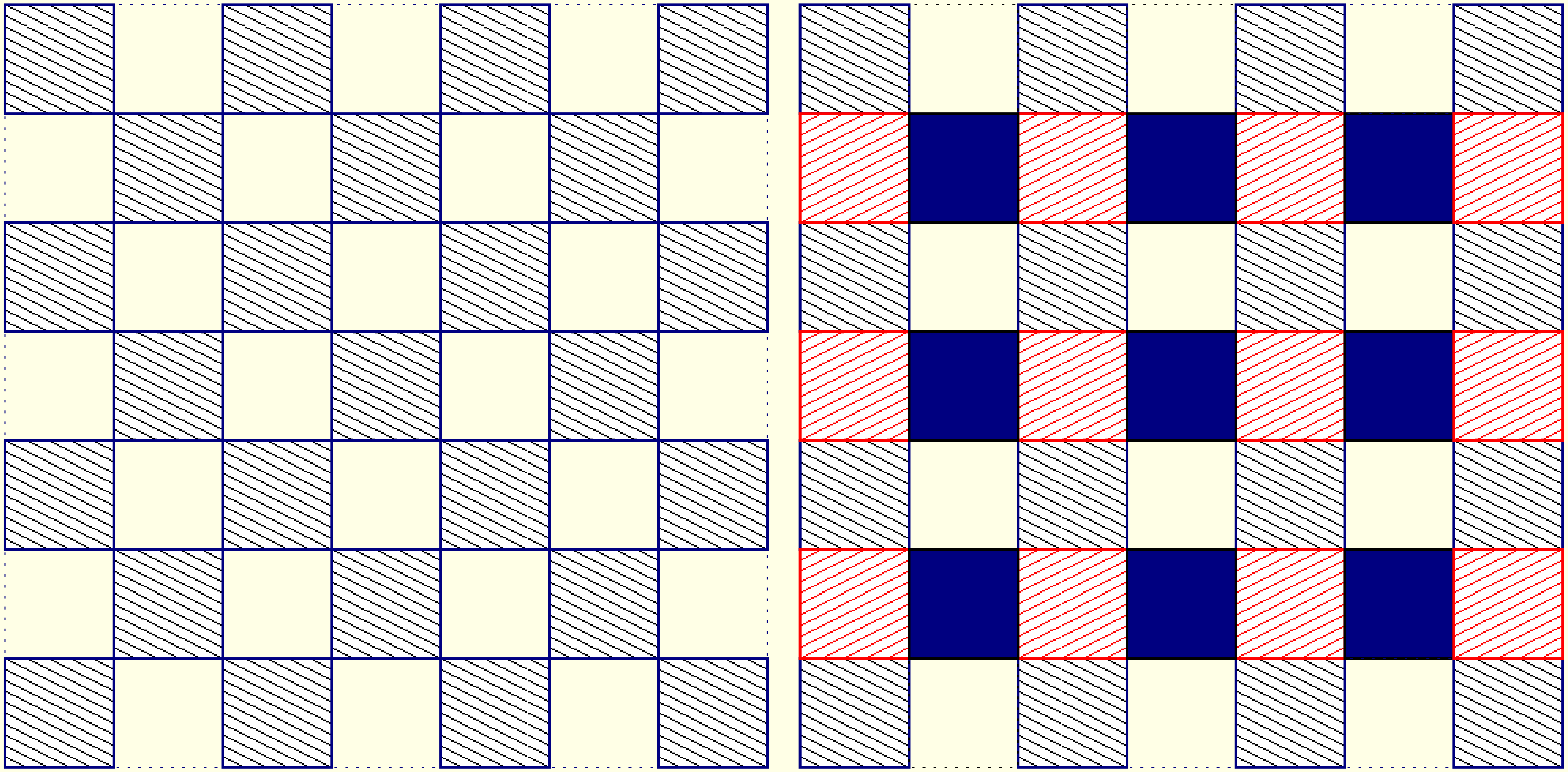
The percolation probability P dependence on the occupation probability p for different values of mixing parameter q which changes from 10% to 90% every 10% from right to left. The N^2 and (N^2+N^3) neighborhoods are mixed. The symbols correspond to $L = 100$ while lines to $L = 500$. The crossing points predict the percolation thresholds $\pi_{23} = \pi_c(q, \text{N}^2, \text{N}^2 + \text{N}^3)$.



percolation probability P
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.4
0.44
0.48
0.52
0.56
0.6
occupation probability p

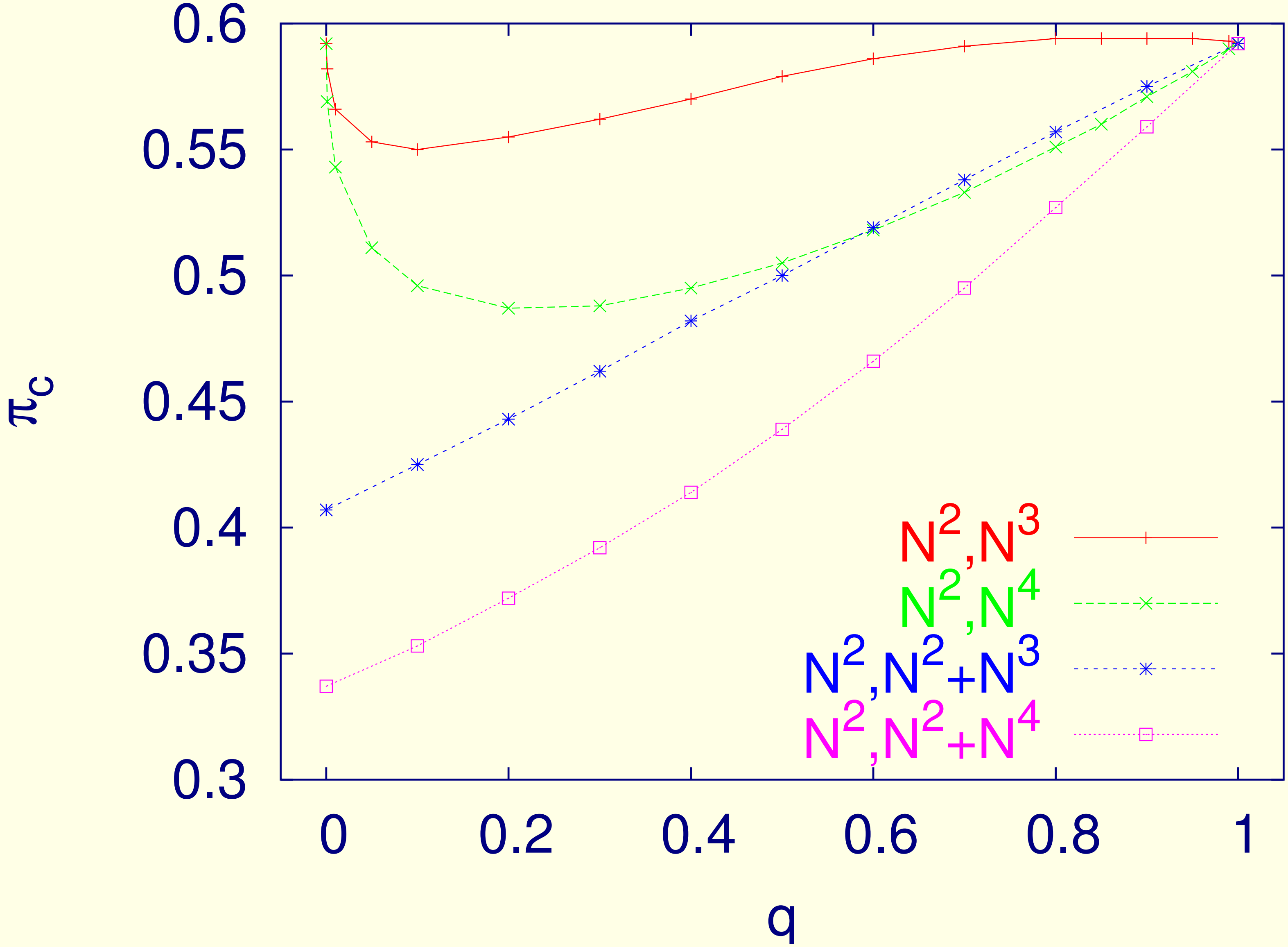
3.1 Percolation thresholds

	z	p_c	p_c^{GM}
N^2	4	$0.592\cdots$	0.5984
N^3	4	$0.592\cdots$	0.5984
N^4	4	$0.592\cdots$	0.5984
N^5	8	$0.298\cdots$	0.4411
N^6	4	$0.592\cdots$	0.5984



	z	p_c	p_c^{GM}
N^2+N^3	8	$0.407\cdots$	0.4411
N^2+N^4	8	$0.337\cdots$	0.4411
N^2+N^5	12	$0.234\cdots$	0.3748
$N^2+N^3+N^4$	12	$0.288\cdots$	0.3748

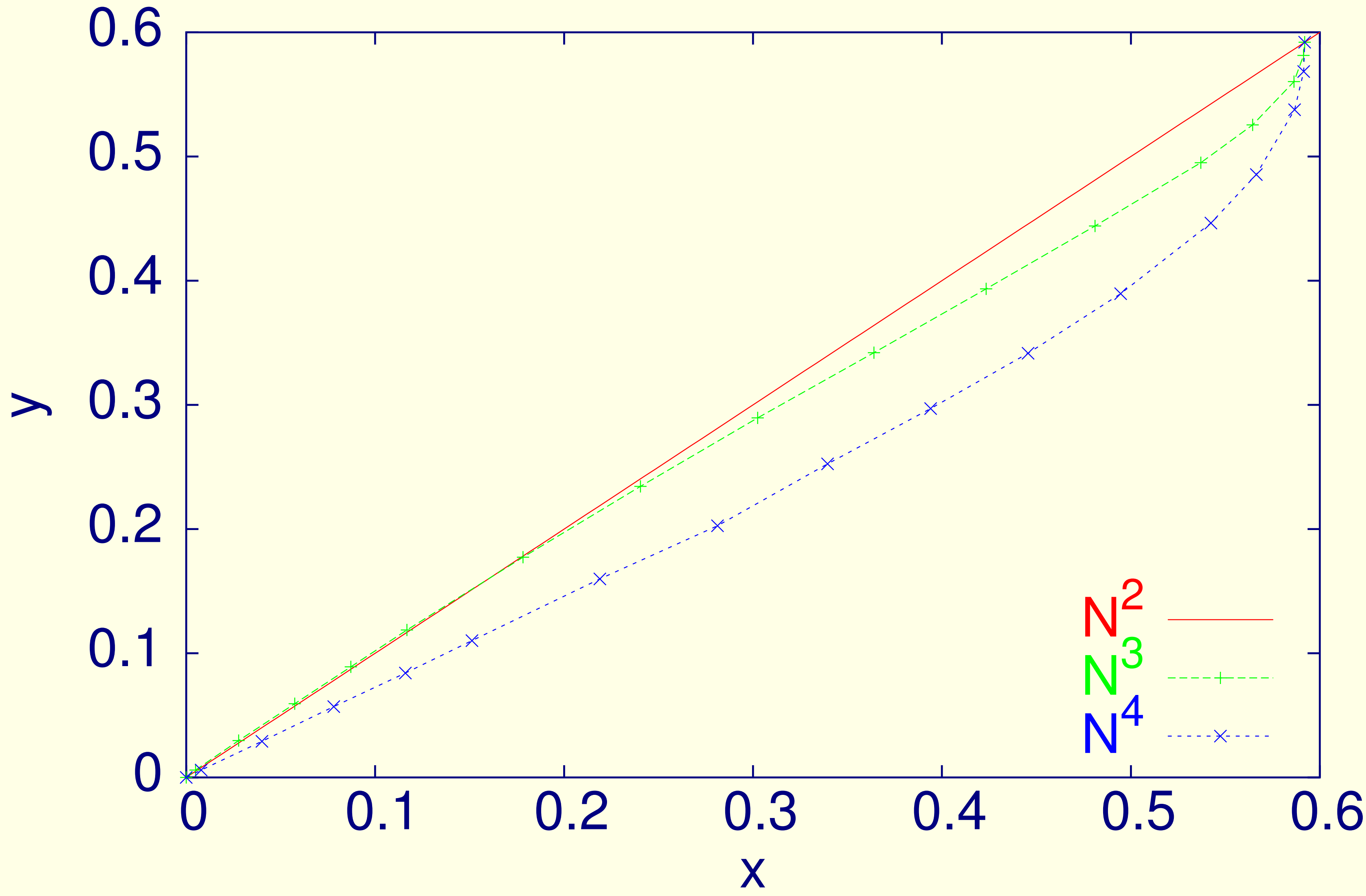
3.2 Restoring sites percolation



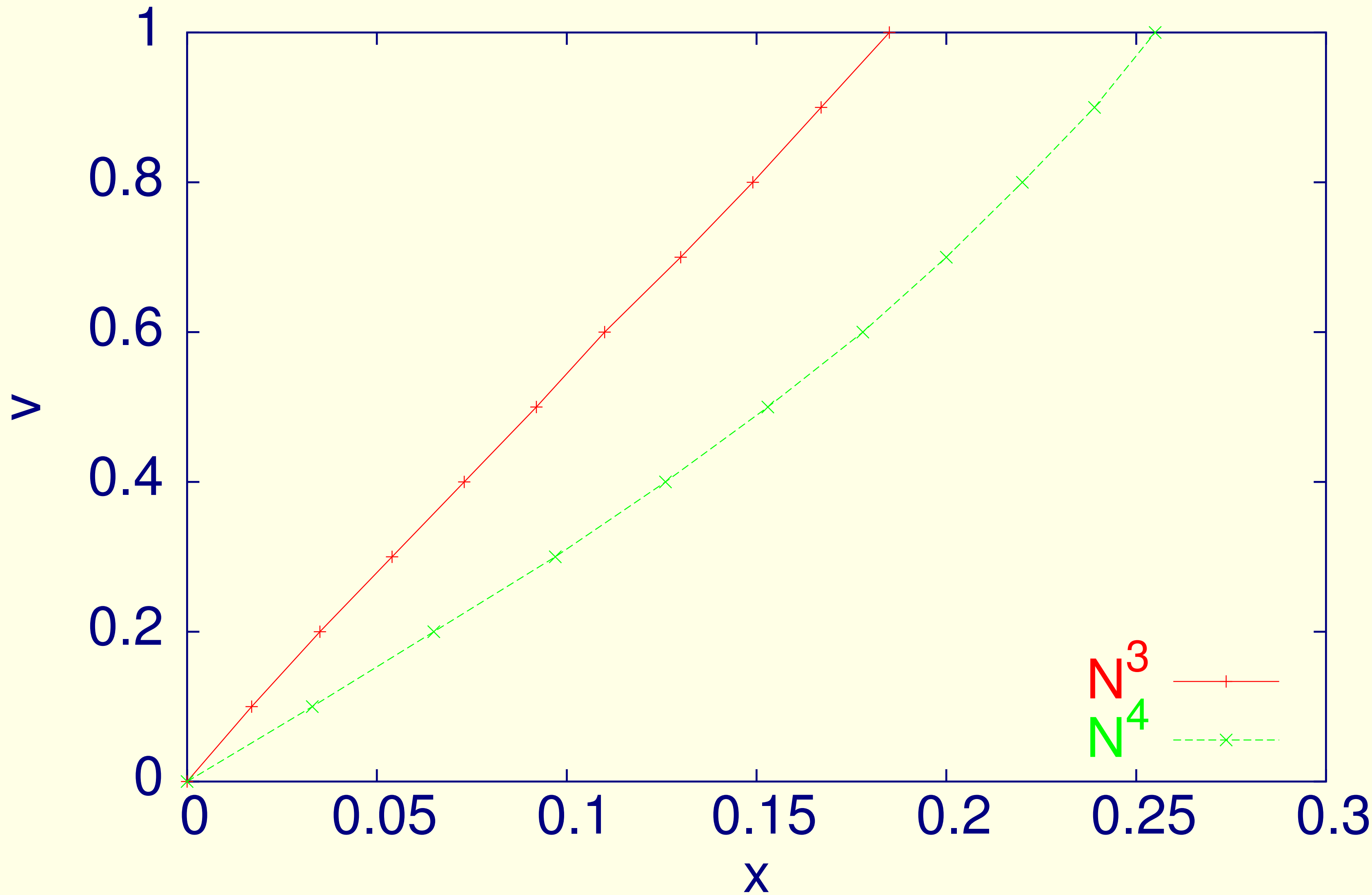
The fraction of sites which must be reoccupied (y) or enriched with long-range links (v) to recover percolation phenomenon when the fraction x of sites with the N^2 was emptied.

$$x = p_c - q\pi_c \text{ and } y_c = (1-q)\pi_c$$

$$x = p_c - \pi_c \text{ and } v_c = (1-q)$$



0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
x
N2
N3
N4



1
0.8
0.6
0.4
0.2
0
v
0
0.05
0.1
0.15
0.2
0.25
0.3
x
N3
N4

4 Conclusions

- Only d and z could not be sufficient to build a universal law $p_c(d,z)$ which extends to complex lattices.
- We have showed quantitatively that restoring requires less sites to be created when longer links are employed for site reconstruction process at intermediate range of damages.

- The strategy involving bond enrichment fails if damages are too large.

- These results may prove useful to some of the large spectrum of physical and interdisciplinary topics where the percolation theory may be applied like forest fires spreading, immunology, liquid migration in porous media, econophysics, and sociophysics.

Acknowledgements

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References

[1] KM, S.Galam, PRE **72**, 016125 (2005)

[2] S.Galam, KM, PRE **72**, 027103 (2005)