Reshuffling spins with short range interactions

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1 Introduction

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- Sociophysics is based on the use of concepts and tools from physics to describe social and political behavior
- While the validity of such a transfer has been long questioned among physicists, none ever has expected that some basic sociophysics question may in turn lead to new development within physics

2 Ising model

$$E \equiv -\frac{1}{2}\sum_{i,j} J_{ij} S_i S_j, \qquad (1)$$

where $S_i = \pm 1$ is the Ising spin variable at each node i

$$J_{ij} = \begin{cases} J > 0 & \text{if } i \text{ and } j \text{ are neighbors,} \\ 0 & \text{otherwise,} \end{cases}$$

is short-range ferromagnetic exchange integral

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- Binder's cumulant $U \equiv 1 - \langle m^4 \rangle / (3\langle m^2 \rangle^2)$, for T_C evaluation is used to avoid finite size effect

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- for each spin i in an initial configuration μ_i, a new configuration η_i resulting from the single spin flip $S_i \longrightarrow -S_i$ is accepted with a probability

$$p^{G}_{\mu_i \to \eta_i} = \frac{\exp(-E_{\eta_i}/k_B T)}{\exp(-E_{\mu_i}/k_B T) + \exp(-E_{\eta_i}/k_B T)}, \tag{2}$$

where E_{η_i} is the energy of configuration η_i,

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- when all N spins are investigated one Monte Carlo step (MCS) is completed

2.2 Metropolis scheme

- the acceptance probability of the new configuration

$$p^{M}_{\mu_i \to \eta_i} = \min\{1, \exp[-(E_{\eta_i} - E_{\mu_i})/k_B T]\} \quad (3)$$

- but here, at each MCS, all the spins are randomly visited and updated

3 The Solomon network [1]

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- in many sociological models, the behavior of each person is influenced by the neighbors, or influences the neighbors
- however, the neighbors in the workplace are different from the neighbors at home

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- in the workplace lattice, the people are numbered consecutively from $i = 1$ to $i = N$ with helical boundary conditions

- the same people also show up on the home lattice, but an different order which is a random permutation of the order on the workplace lattice

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- a simplified version of this two-lattice model uses only one lattice, and defines the neighborhood of each site as being the nearest neighbors plus one randomly selected site anywhere in the lattice

- for a chain the neighbors of i are thus $i \pm 1, i + R$ where R is a random distance; these neighbors can be compared with those of the honeycomb lattice: $i \pm 1, i + L$

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- therefore it cannot be excluded that **ordering** is found also in **one dimension**

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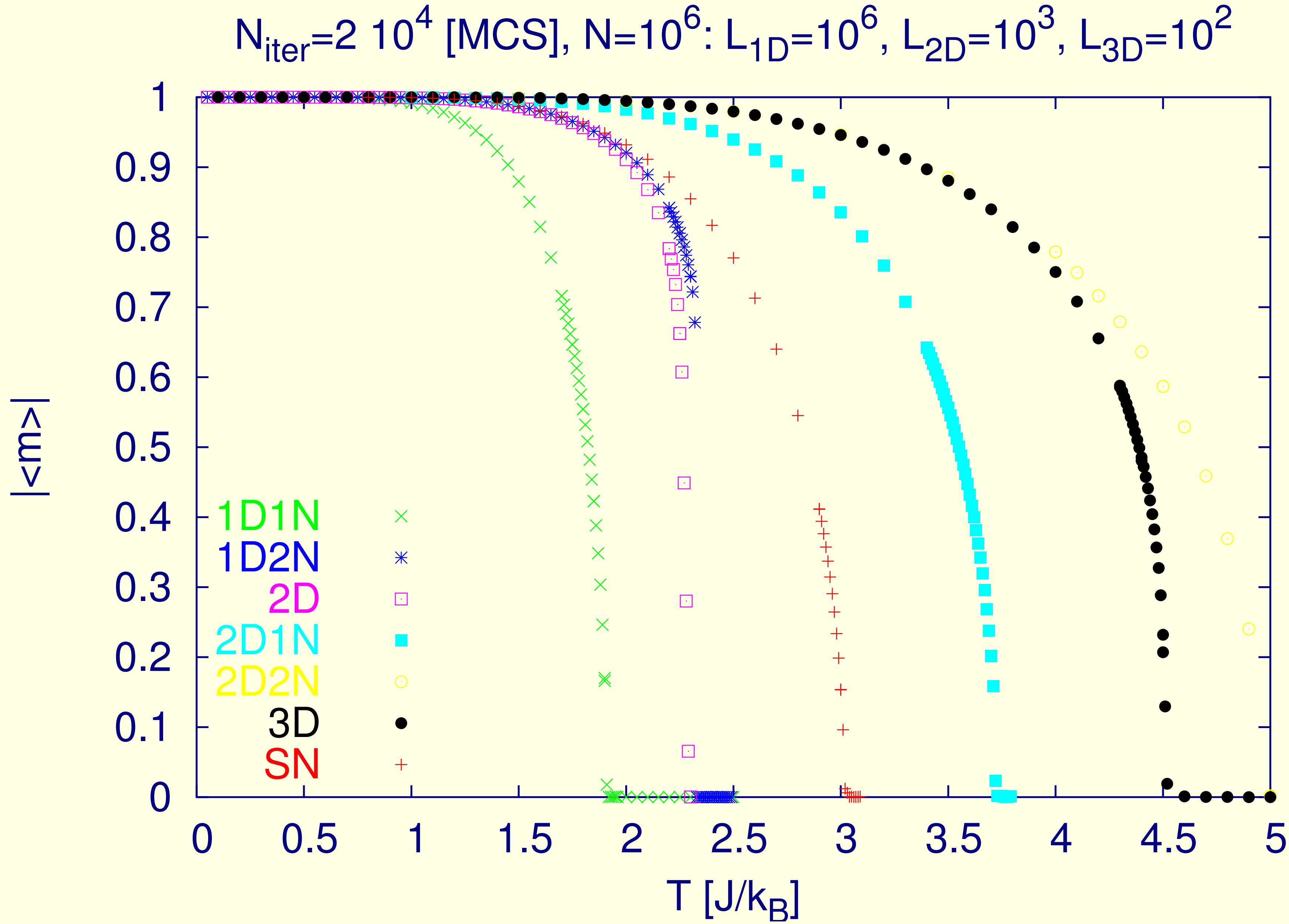
- the simulation is performed on $N = 10^6$ sites with helical boundary conditions and initially, at $t = 0$, all spins are pointing up ($S_i = +1$ for $i = 1, 2, \ldots, N$)
- m is averaged over the last 10^4 MCS



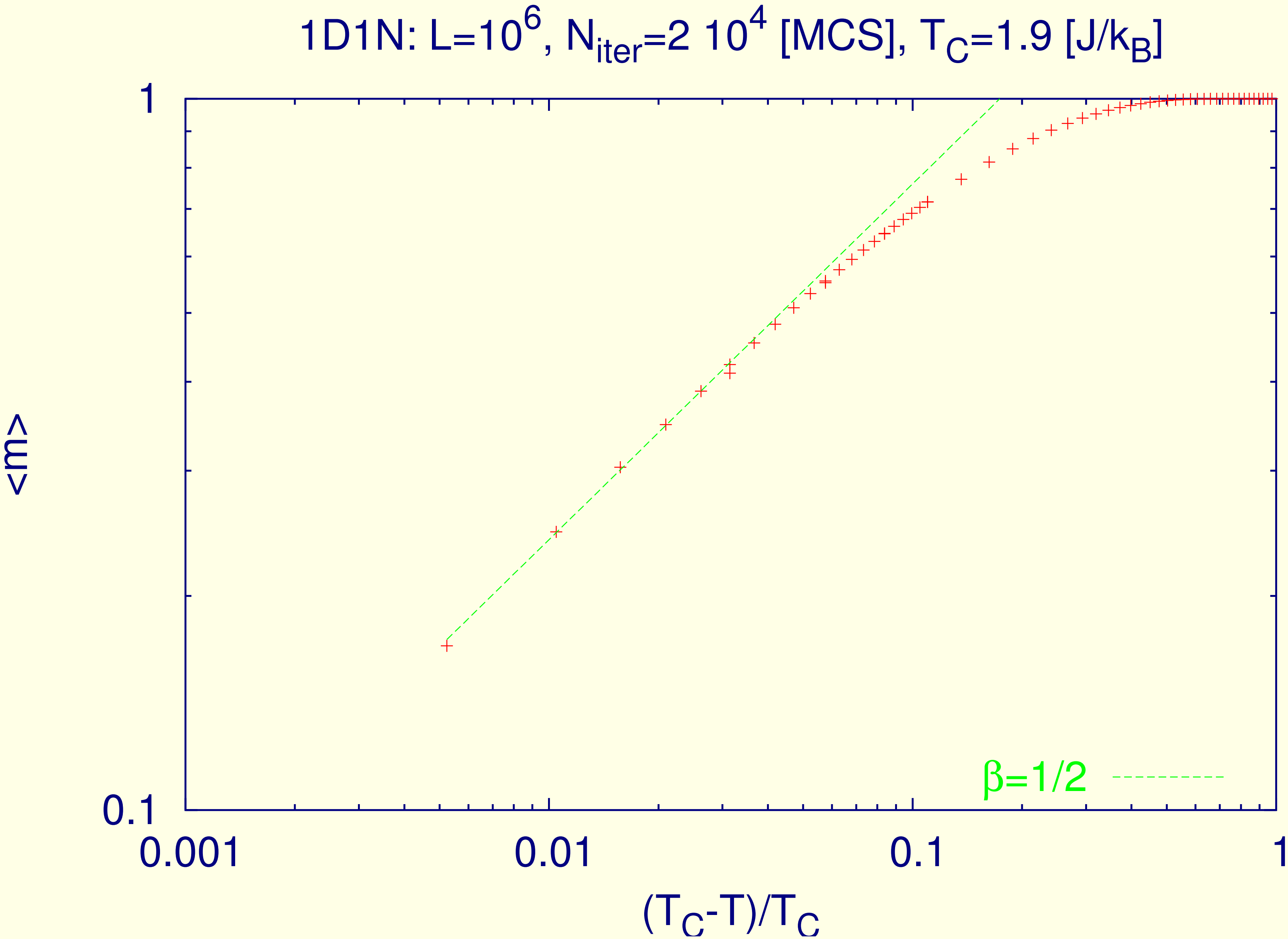
N_{iter}=2 10^4 [MCS], N=10^6: L_{1D}=10^6, L_{2D}=10^3, L_{3D}=10^2
|<m>|
T [J/k_B]
1D1N
1D2N
2D
2D1N
2D2N
3D
SN

- introducing one additional neighbor somewhere at the lattice in 1D case shifts the "social Curie point" from $T_C = 0$ towards $T_C \approx 1.9(1)\ [J/k_B]$

- for the 1D1N case:

- we have found the critical exponent β (describing the critical behavior of the magnetization in the vicinity of the transition) equal to $1/2$

- we have observed that the average absolute value of the magnetization $\langle |m| \rangle$ for $T \approx T_C$ decreases with the system size $10 \leq L \leq 10^4$



1D1N: L=10^6, N_iter=2 10^4 [MCS], T_C=1.9 [J/k_B]
1
0.1
0.001
0.01
0.1
1
<m>
(T_C-T)/T_C
β=1/2

4 GRIM, SLIM, TRIM [2]

- **G**radually **R**eshuffled **IM**: in every MCS, before updating all spins, with a probability p the reshuffling procedure takes place: random permutation of all spin labels is produced and their positions are rearranged according to that new labels order
- with probability $1 - p$, all spins keep their current position

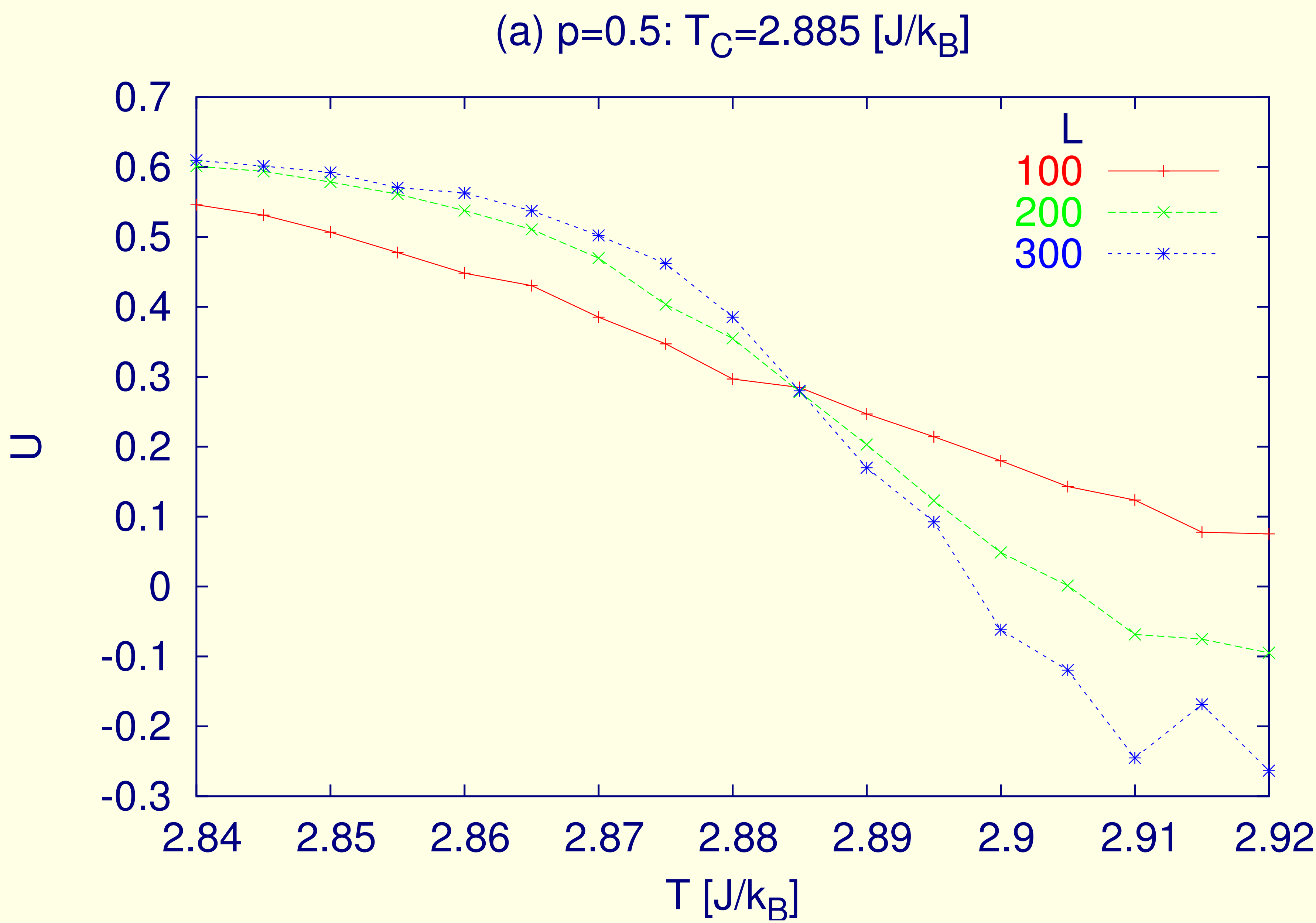
- **S**quare **L**attice **IM** ($p = 0$)
- **T**otally **R**eshuffled **IM** ($p = 1$) — Galam model [4]

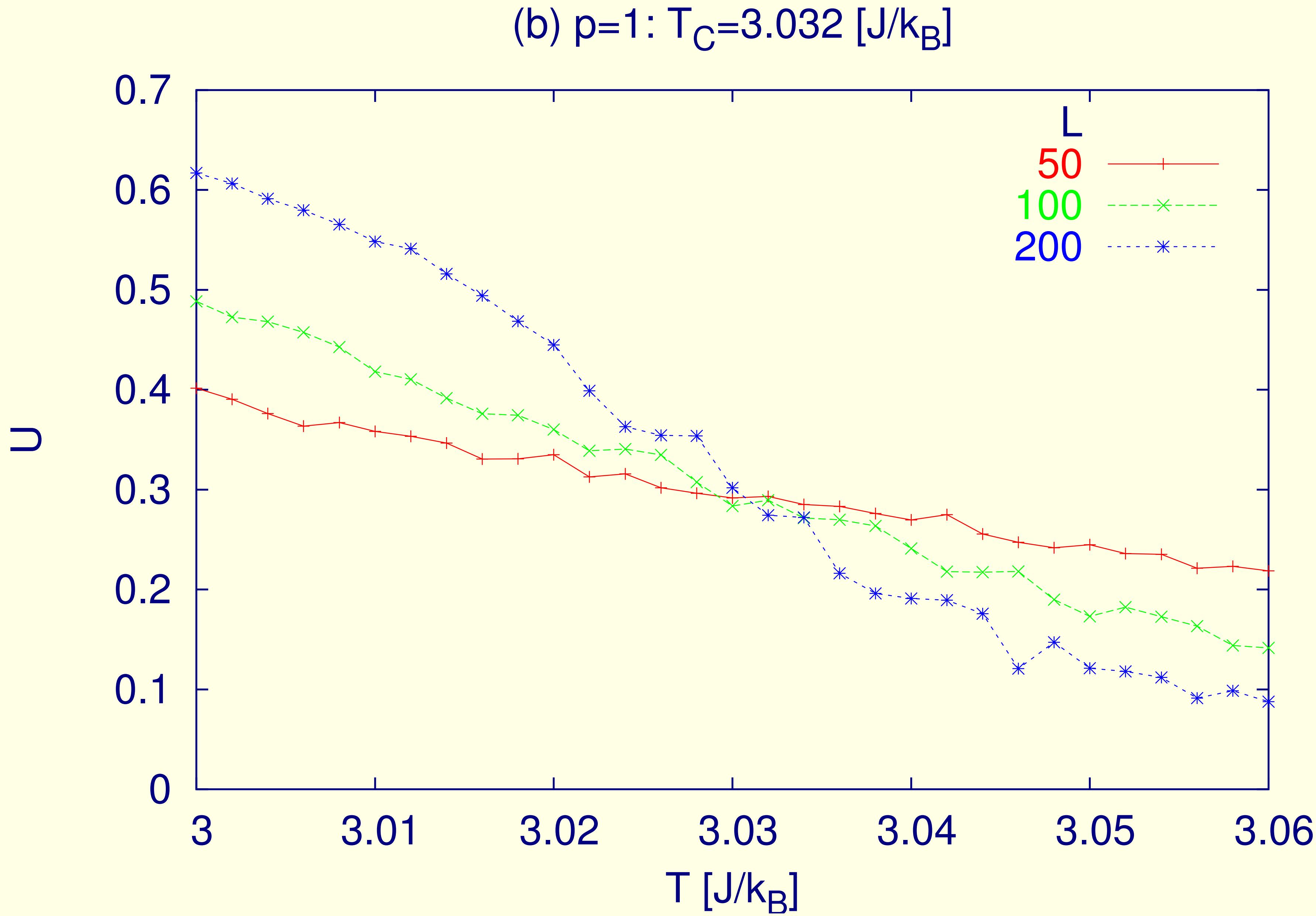
4.1 Results

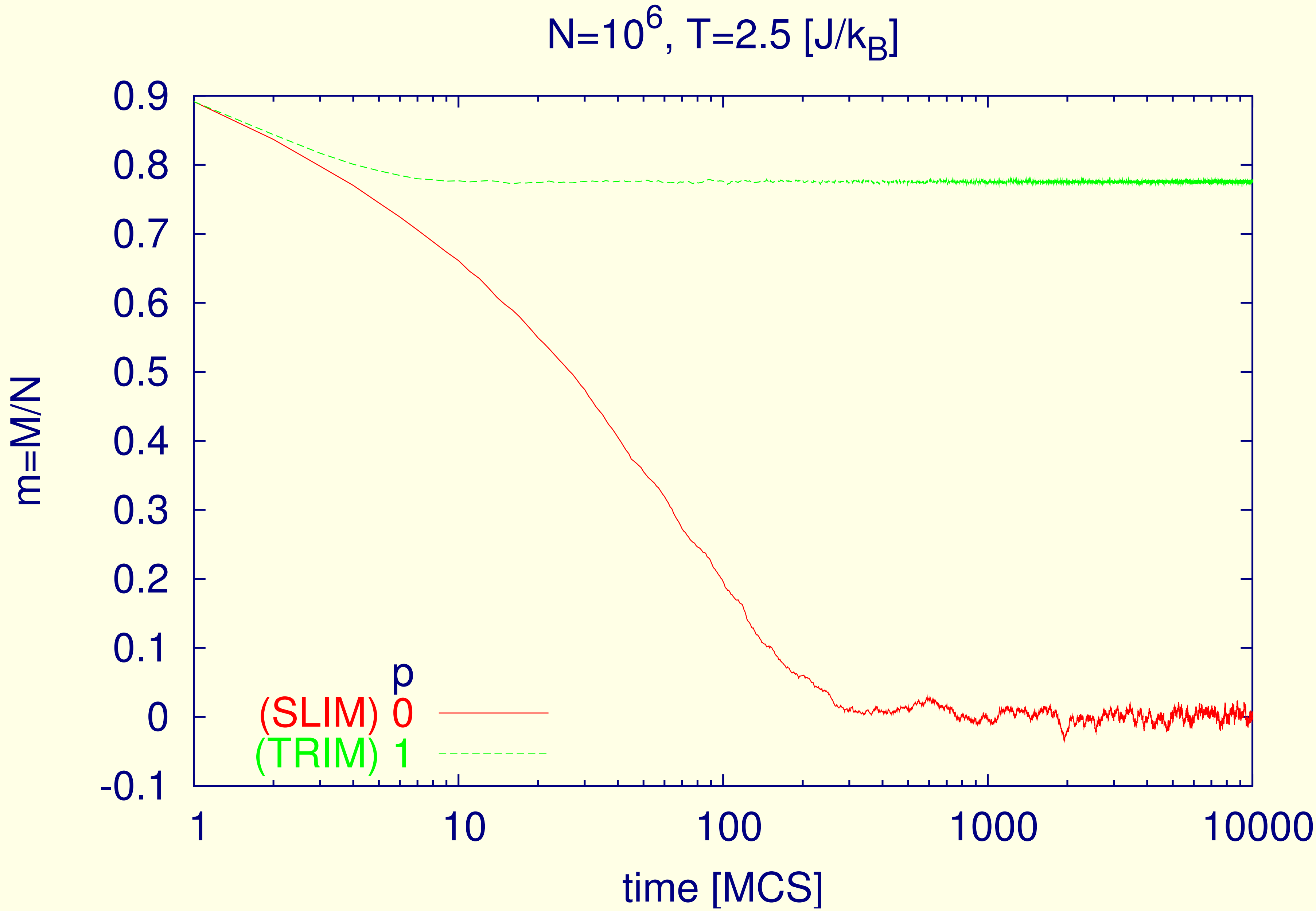
- m vs $T \longrightarrow T_C$

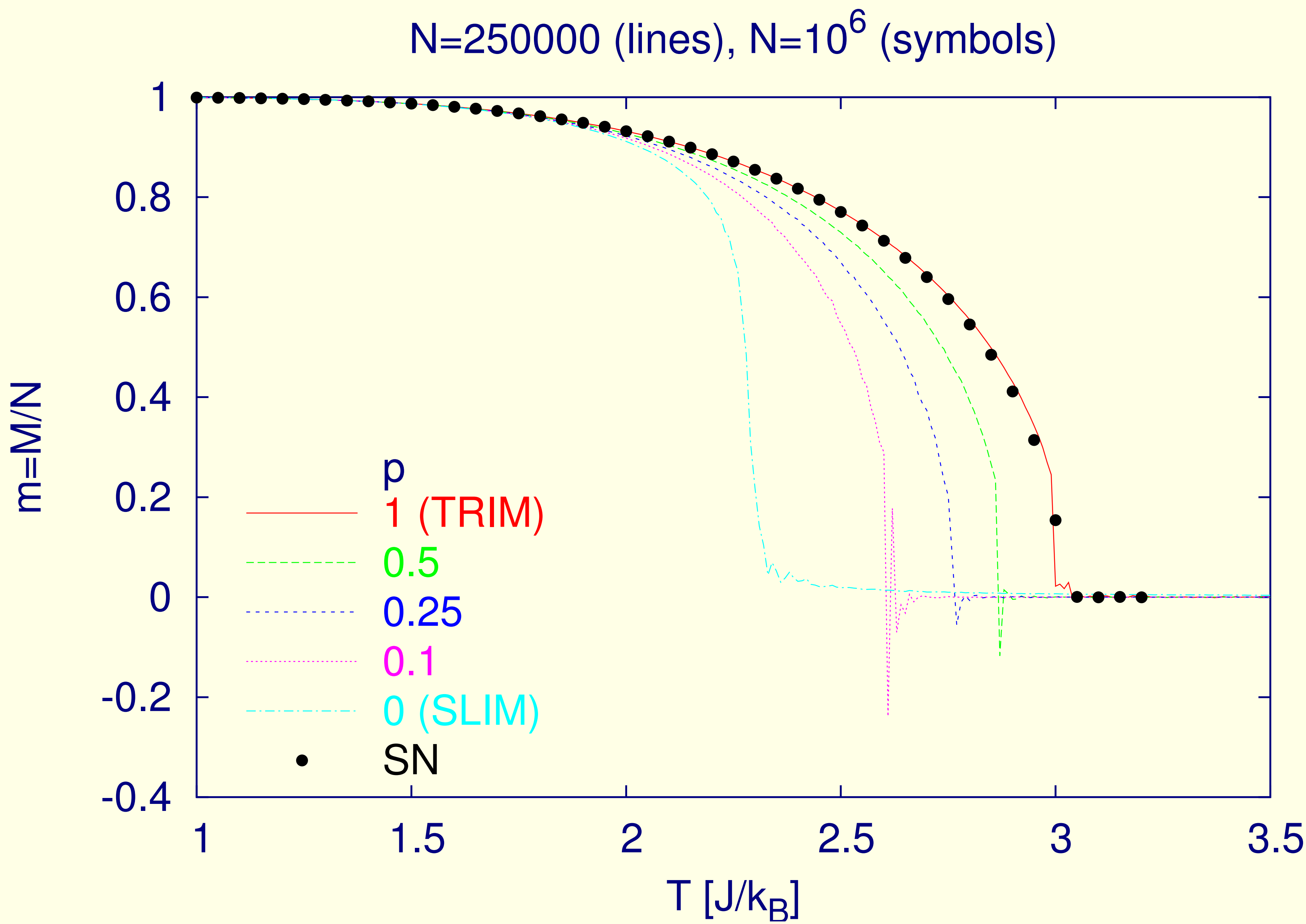
- U vs $(T, N) \rightarrow T_C$, to get a more precise estimate of T_C in the thermodynamic limit (taken over $4 \cdot 10^5$ MCS after discarding 10^5 MCS for equilibration at each T)

- U should go to $2/3$ below T_C and to zero above T_C when the size increases, and the finite-size estimates are expected to cross around T_C

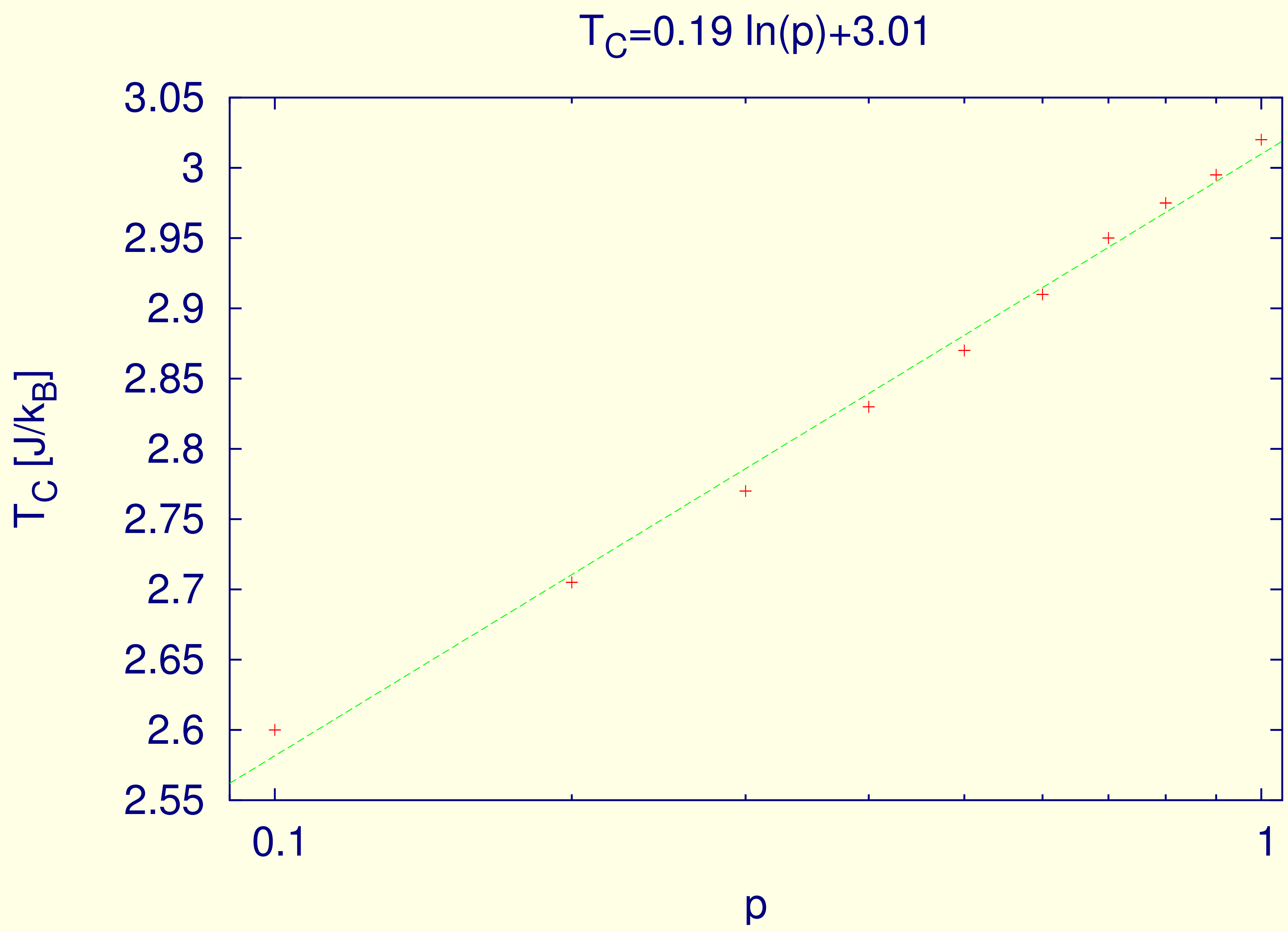

(a) p=0.5: T_C=2.885 [J/k_B]
L
100
200
300
U
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
2.84
2.85
2.86
2.87
2.88
2.89
2.9
2.91
2.92
T [J/k_B]


(b) p=1: T_C=3.032 [J/k_B]
L
50
100
200
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
U
3
3.01
3.02
3.03
3.04
3.05
3.06
T [J/k_B]

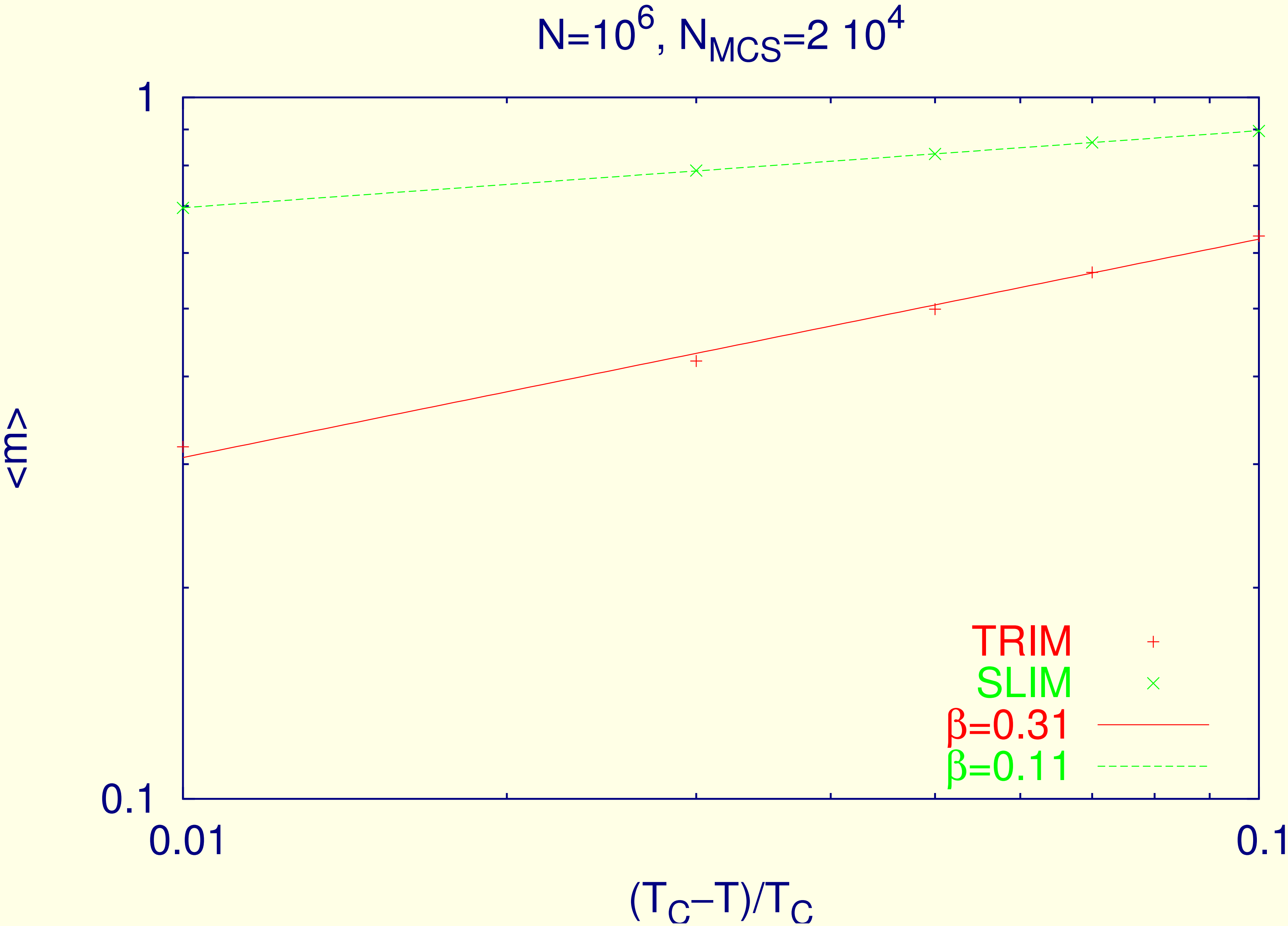

N=10^6, T=2.5 [J/k_B]
m=M/N
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
1
10
100
1000
10000
time [MCS]
p
(SLIM) 0
(TRIM) 1



N=250000 (lines), N=10^6 (symbols)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
m=M/N
p
1 (TRIM)
0.5
0.25
0.1
0 (SLIM)
SN
1
1.5
2
2.5
3
3.5
T [J/k_B]



T_C=0.19 ln(p)+3.01
3.05
3
2.95
2.9
2.85
2.8
2.75
2.7
2.65
2.6
2.55
T_C [J/k_B]
0.1
1
p



$N=10^6$, $N_{MCS}=2\ 10^4$
1
0.1
0.01
0.1
<m>
$(T_C-T)/T_C$
TRIM
SLIM
β=0.31
β=0.11

4.2 The Galam unifying scheme [3]

- we now go one step further in the investigation of the validity of Galam unifying scheme

- it is a general sequential frame, which operates via local updates of small groups of spins randomly selected

• if $p(t)$ is the proportion of plus spins at time t, we obtain for the new proportion $p(t+1)$ after one update cycle

$$p(t+1) = \sum_{k=0}^{5} \binom{5}{k} g_k [p(t)]^k [1-p(t)]^{5-k}, \quad (4)$$

where g_k as the probability that a group of five spins with k plus and $(5-k)$ minus ends up with five plus

- from up-down symmetry this number reduces to three with $g_0 = 1 - g_5$, $g_1 = 1 - g_4$ and $g_2 = 1 - g_3$

- we can calculate the unstable fixed points from Eq. (4) to get the corresponding critical temperature

- the corresponding energies E_{μ_i} are calculated as well as energies E_{η_i} are obtained once the central spin has been flipped

4.2.1 Metropolis dynamics

$$g_5 = \frac{5-d}{5}, g_4 = \frac{21-4c}{25}, g_3 = \frac{28}{50},$$

where

$$c = \exp\left(-4J/k_B T\right), \qquad d = \exp(-8J/k_B T)$$

4.2.2 Glauber dynamics

$$g_5 = \frac{4+a}{5}, g_4 = \frac{20+a-4b}{25}, g_3 = \frac{31-4b}{50},$$

where

$$a = \frac{\exp(4J/k_BT)}{\exp(4J/k_BT)+\exp(-4J/k_BT)}$$

$$b = \frac{\exp(-2J/k_BT)}{\exp(2J/k_BT)+\exp(-2J/k_BT)}$$

5 Conclusions

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5 Conclusions

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- $T_C = 1.59\,[J/k_B]$ for Metropolis scheme
- the Glauber result is rather close to the numerical finding $T_C = 3.03\ [J/k_B]$
- T_C a little bit larger than $3\ [J/k_B]$ was obtained previously by Malarz from a Monte Carlo simulation of SN [1]

- Galam reshuffling [4] is neither a mean-field treatment nor the usual Ising model (SLIM). In particular it yields non mean field exponents $0.31 \approx \beta(TRIM) \neq \beta(MF) = 1/2$

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- does reshuffling create a new universality class for the Ising model?

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- does reshuffling create a new universality class for the Ising model?

- at which value of the reshuffling parameter p does the crossover occur?

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[3] S.Galam, Eur. Phys. Lett. **70** (2005) 705.

[4] S.Galam, J. Math. Psychol. **30** (1986) 426.