Lokalizacja w jednowymiarowych układach ze skorelowanym nieporządkiem

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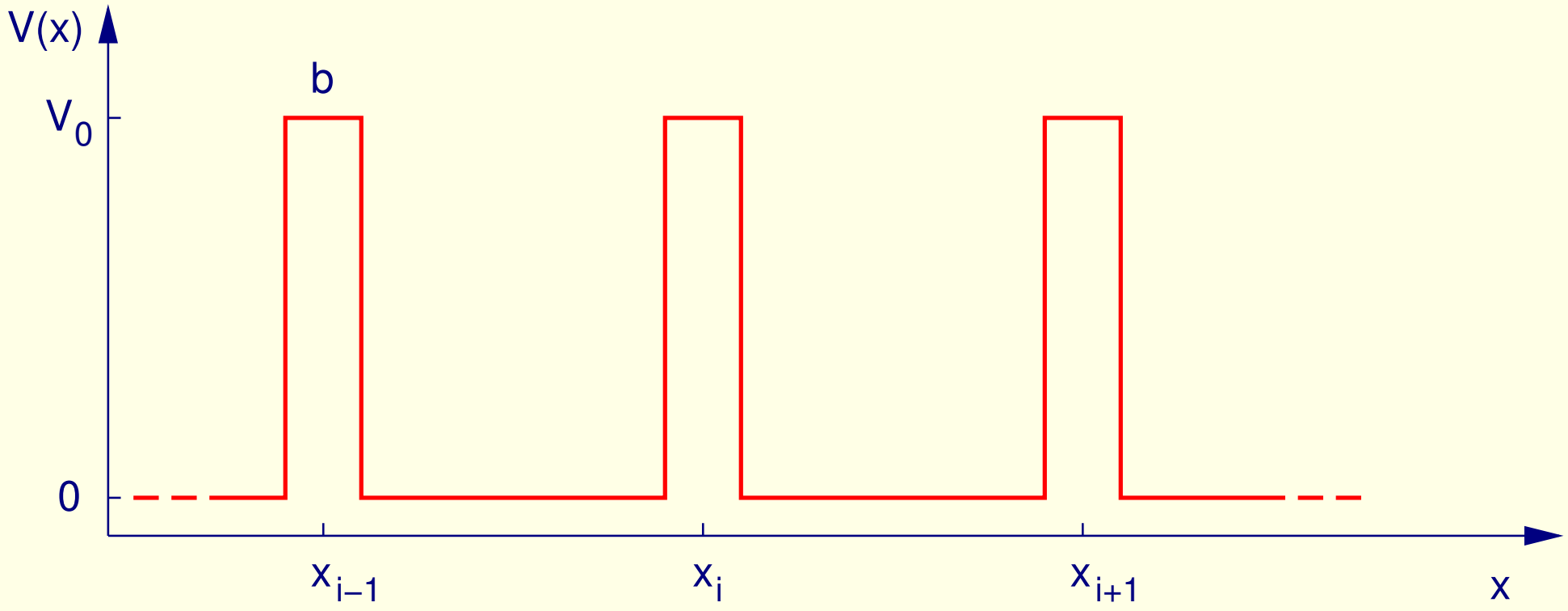
8 maja 2006

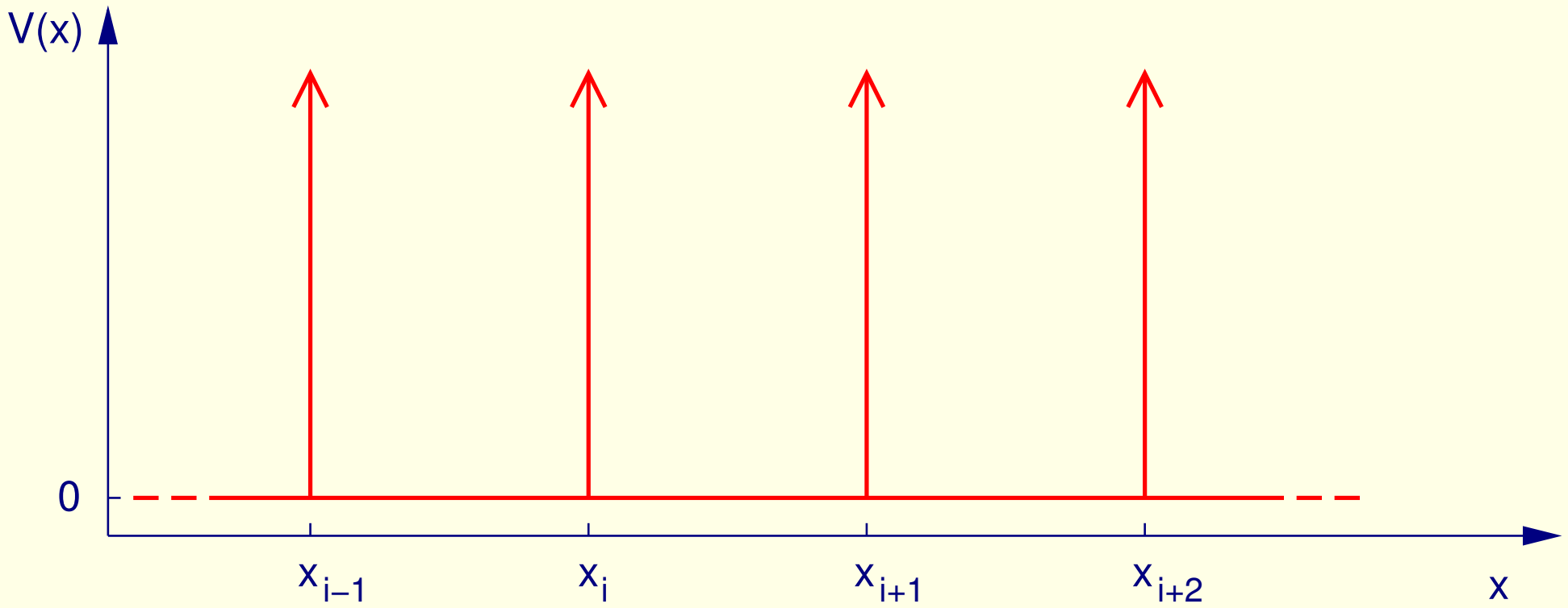
1 Model

- Oparty na modelu Kroniga-Penneya: ciąg prostokątnych barier potencjału



... przejście graniczne

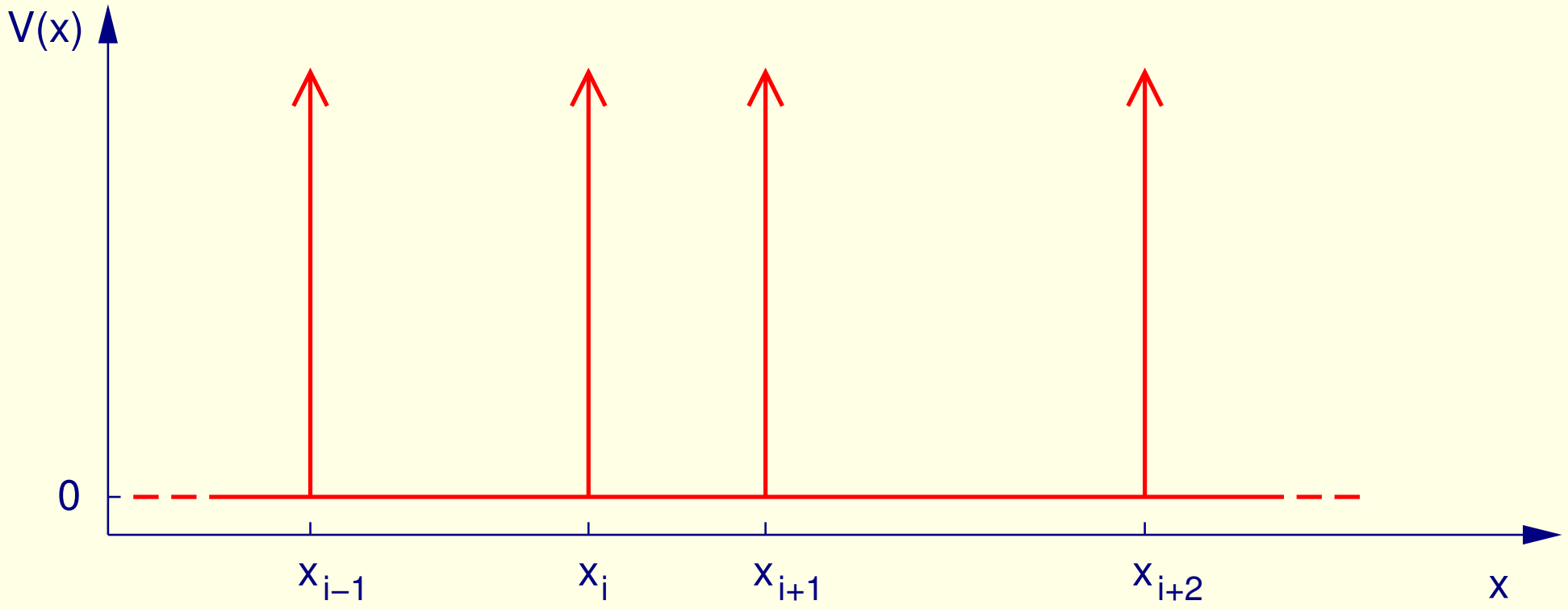
$$b \to 0\,, \quad V_0 \to \infty$$



... modyfikacja dopuszczająca przypadkowe położenia x_i

$$V(x) = V_0 b \sum_i \delta(x - x_i)$$

$\longrightarrow$ "Liquid Kronig-Penney" (η – par. nieporządku)



Możliwe rozwiązywanie równania Schrödingera dla użytego modelu:

- rozwiązania w poszczególnych studniach potencjału wyznaczone analitycznie,
- sklejenie rozwiązań w węzłach x_i (pochodna ma skok),
- warunki brzegowe $\psi(0) = \psi(L) = 0$

2 Ciągi quasi-periodyczne – wybrane przykłady

N. J. A. Sloane, (2006), The On-Line Encyclopedia of Integer Sequences,
www.research.att.com/~njas/sequences/

2.1 Fibonacci

[Sloane A003849]

$$0 \longmapsto 01,$$

$$1 \longmapsto 0$$

liczba zer w n-tym wygenerowanym tak ciągu $= F_n$

(liczba Fibonacciego, $F_n = F_{n-1} + F_{n-2}$)

0 1 0 0 1 0 1 0 0 1 0 0 1 0 1 0 0 1 0 1 0..

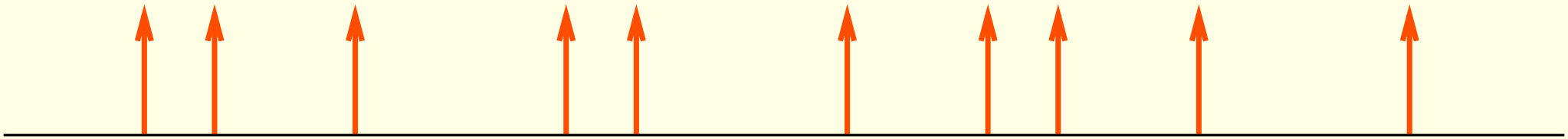
2.2 Thue-Morse

[Sloane A010060]

$$0 \longmapsto 01,$$

$$1 \longmapsto 10$$

0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0 1 0 0 1 0..



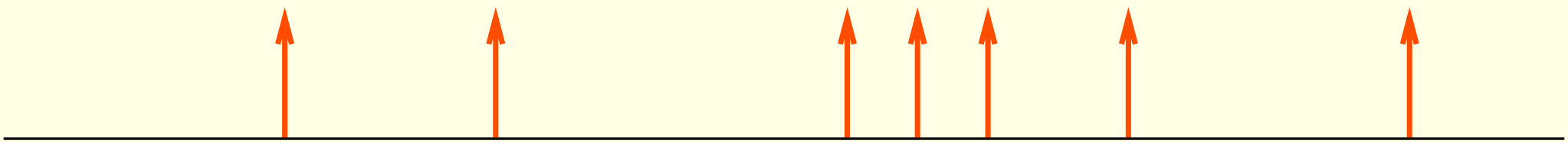
2.3 Rudin-Shapiro (Golay-Rudin-Shapiro)

[Sloane A020987]

$$00 \longmapsto 0001,\ 01 \longmapsto 0010,$$

$$10 \longmapsto 1101,\ 11 \longmapsto 1110$$

0 0 0 1 0 0 1 0 0 0 0 1 1 1 0 1 0 0 0 1 0..



2.4 „Period-doubling”

[Sloane A096268]

$$0 \longmapsto 01,$$

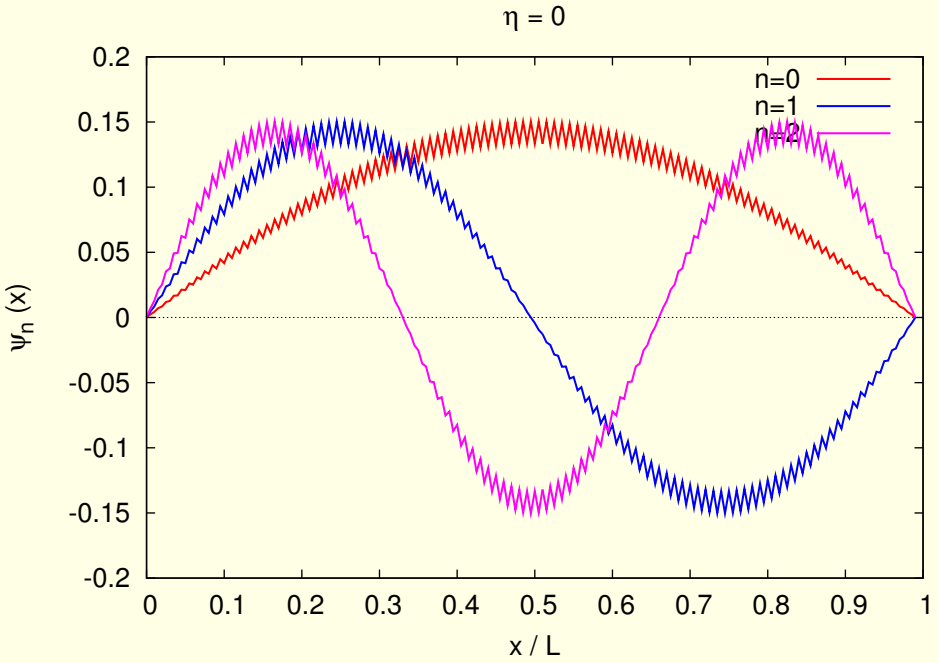
$$1 \longmapsto 00$$

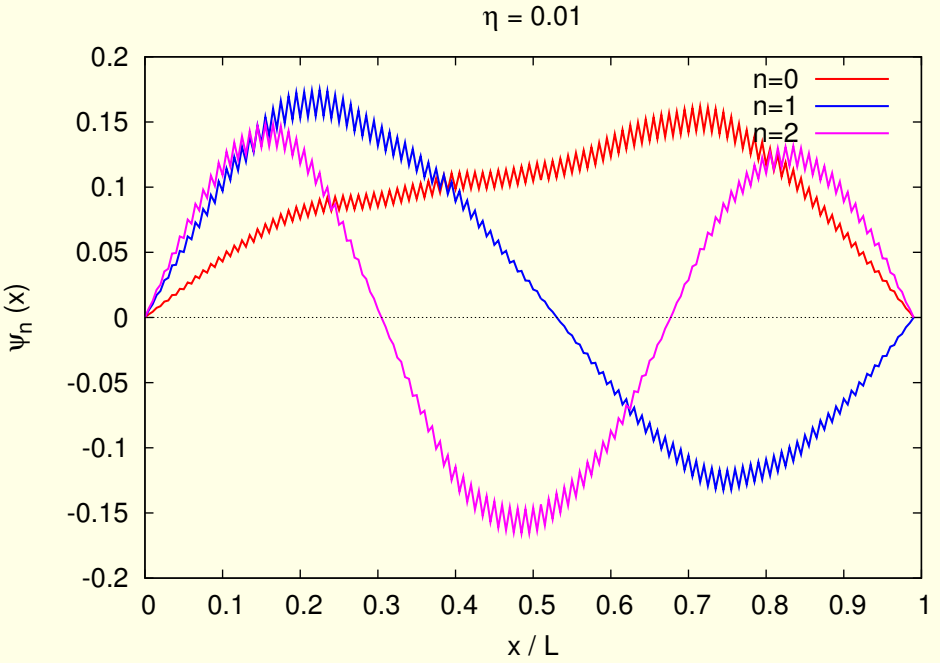
0 1 0 0 0 1 0 1 0 1 0 0 0 1 0 0 0 1 0 0 0..

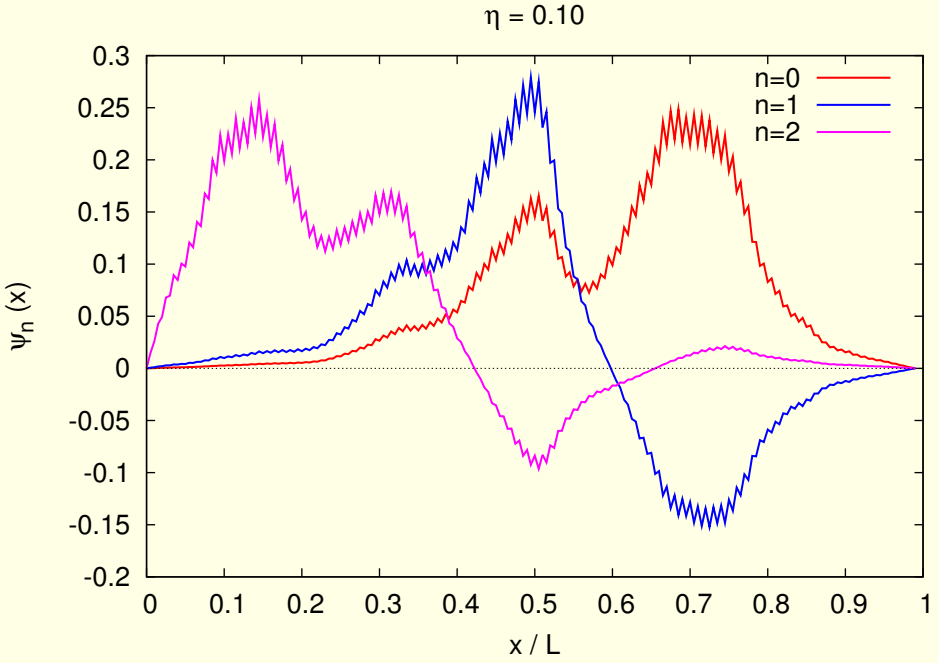
3 Lokalizacja funkcji falowej

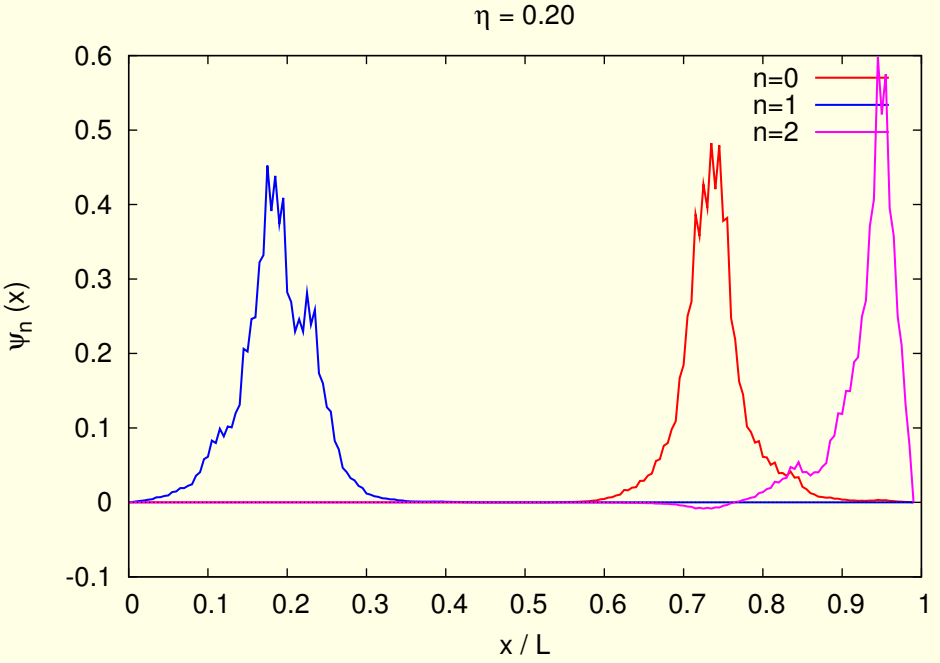
Definicja długości lokalizacji λ:

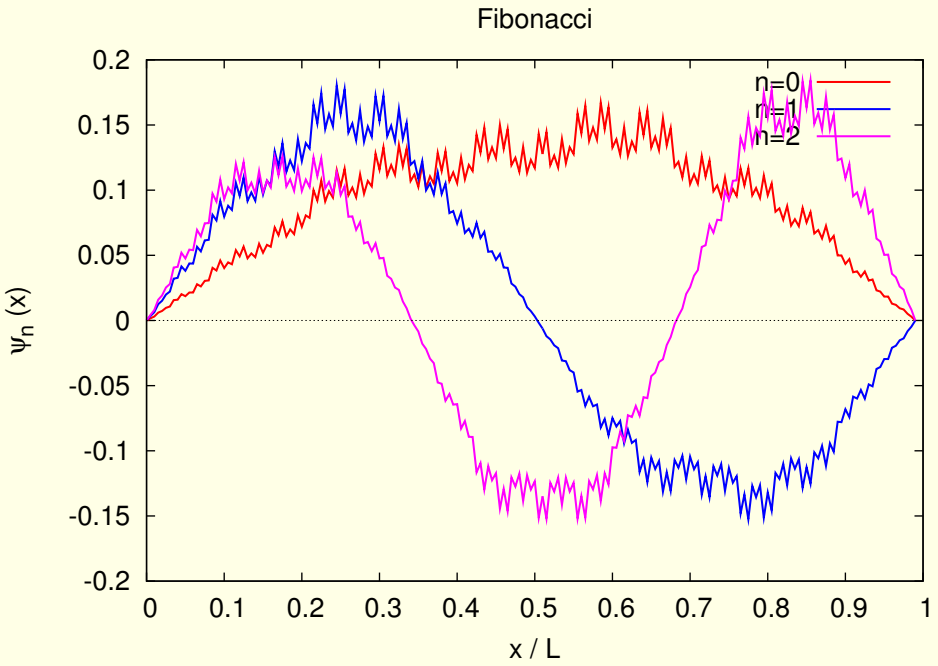
$$|\psi_n(x)| = A(x)\exp(\frac{|x-x_0|}{\lambda})$$

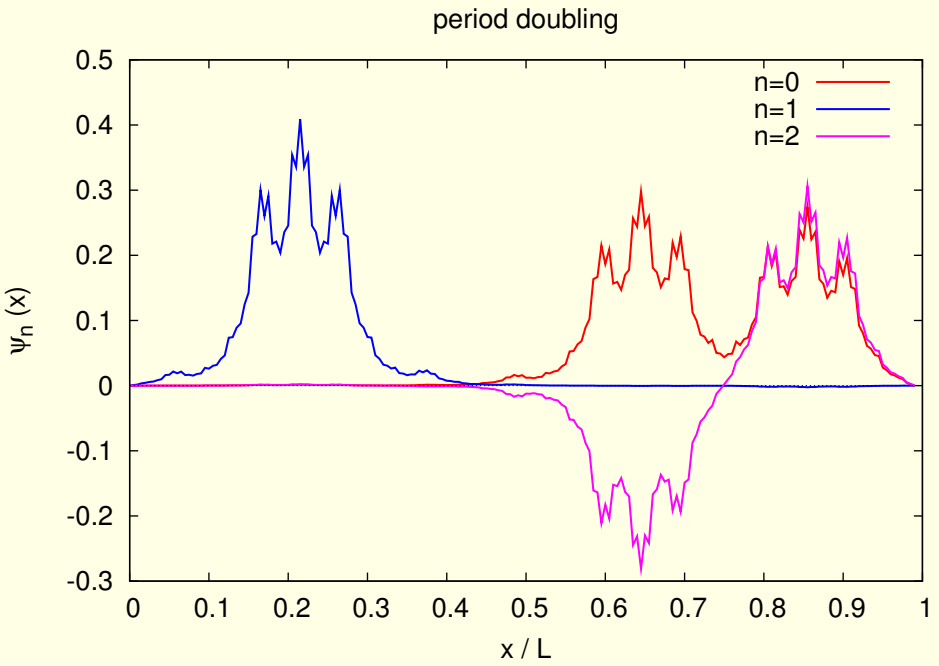

η = 0
n=0
n=1
n=2
ψn (x)
x / L

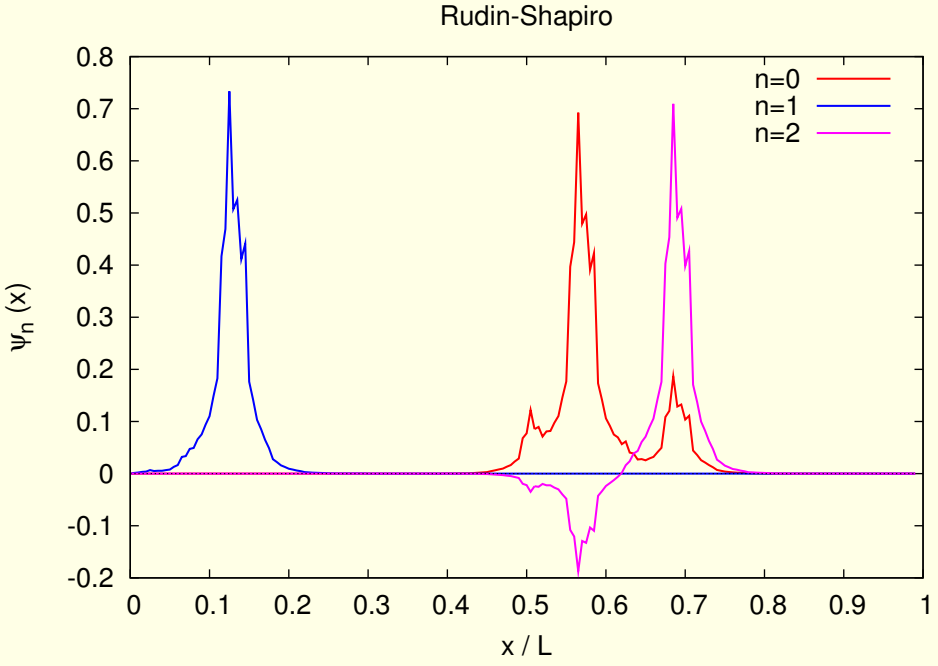

η = 0.01
n=0
n=1
n=2
ψn (x)
x / L

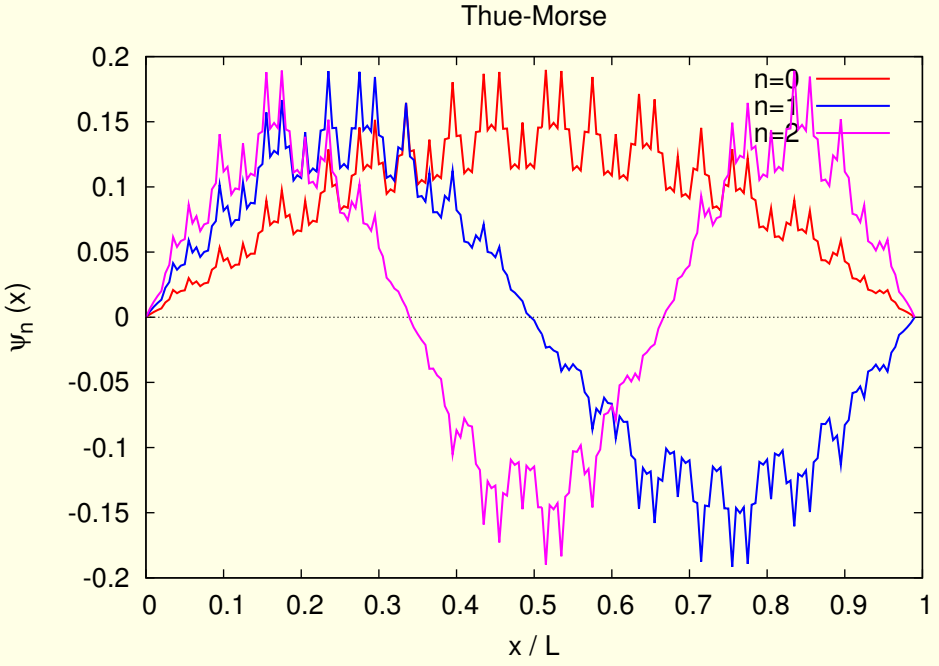

η = 0.10
n=0
n=1
n=2
ψn (x)
x / L


η = 0.20
n=0
n=1
n=2
ψn (x)
x / L


Fibonacci
n=0
n=1
n=2
ψn (x)
x / L


period doubling
n=0
n=1
n=2
ψn (x)
x / L

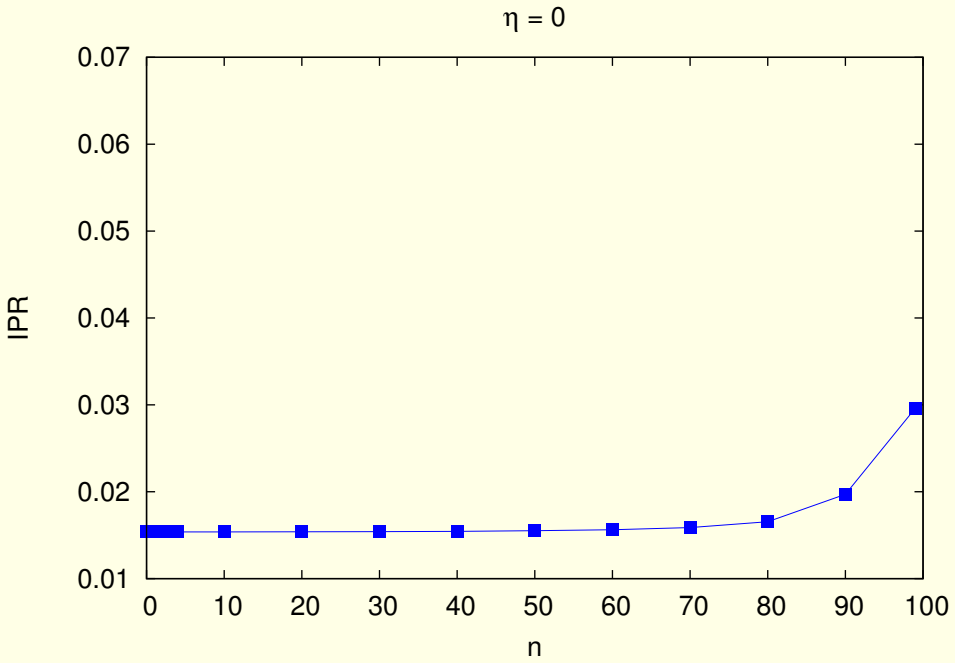

Rudin-Shapiro
n=0
n=1
n=2
ψn (x)
x / L

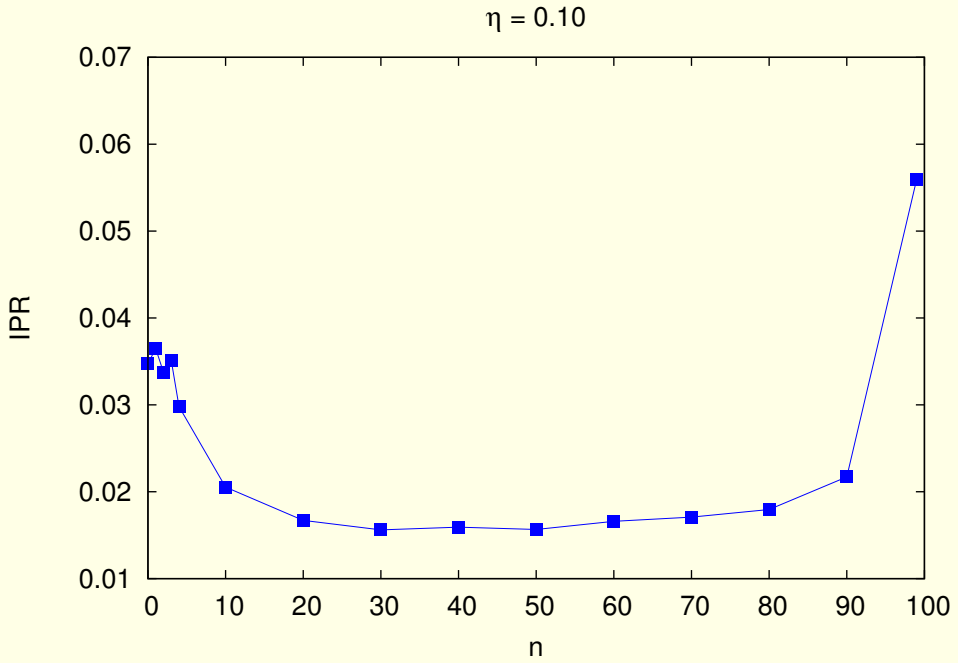

Thue-Morse
n=0
n=1
n=2
ψn (x)
x / L

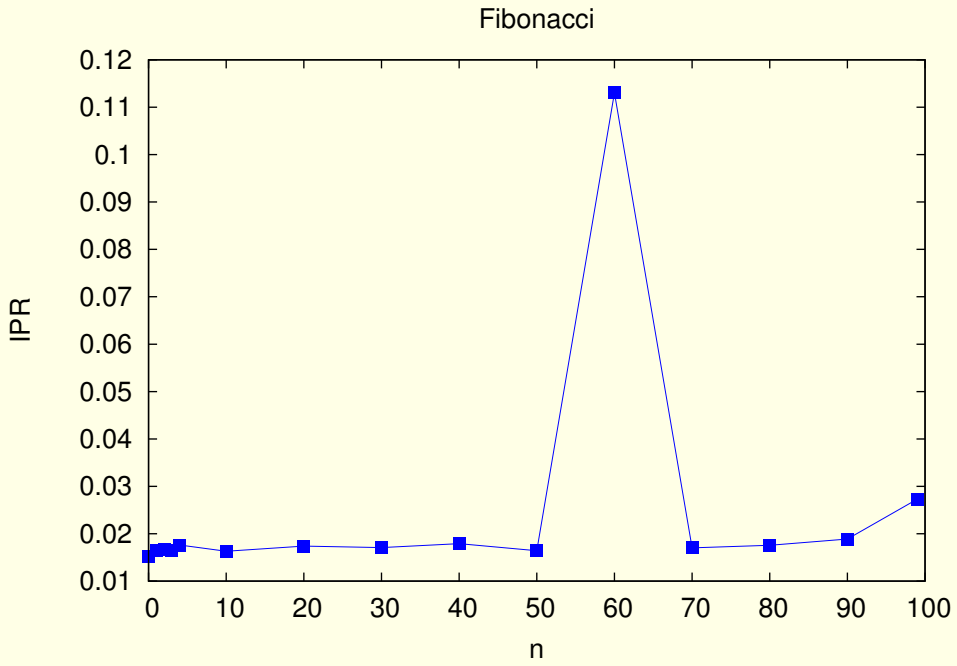
Miara lokalizacji:

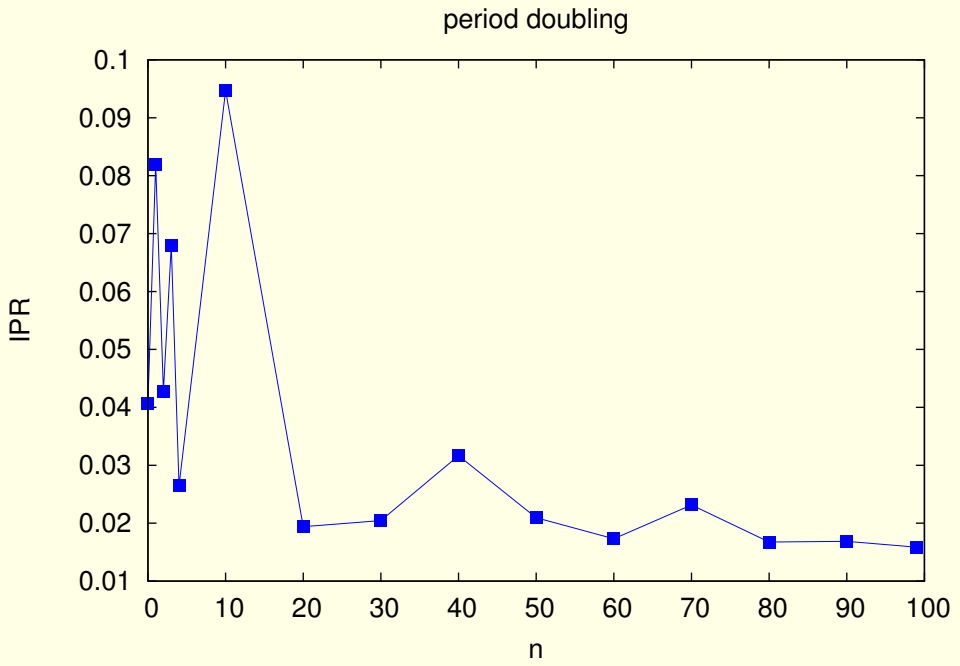
– parametr IPR (Inverse Participation Ratio)

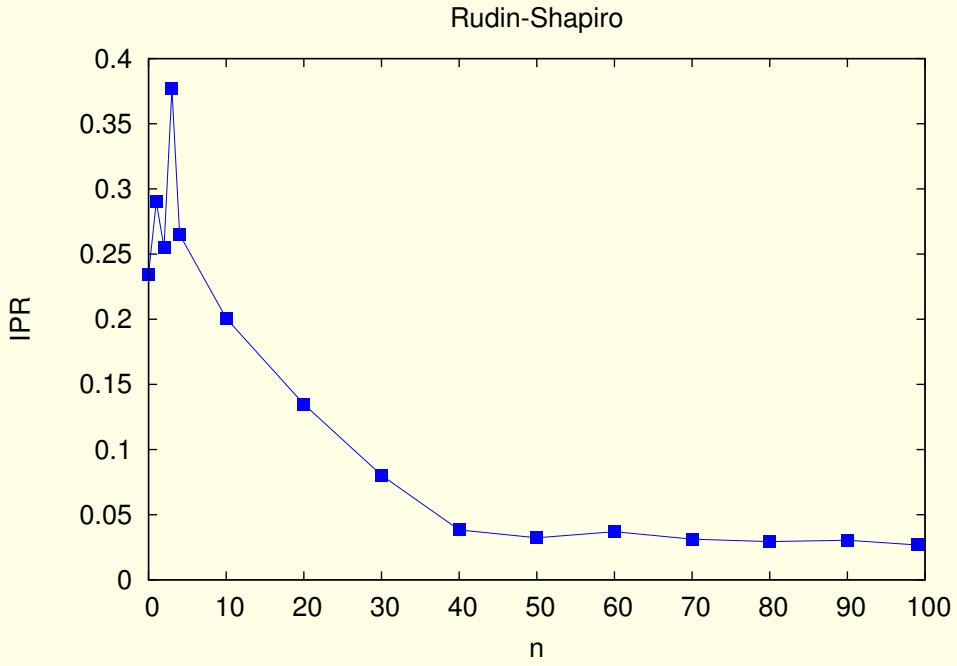
$$P_n^x = \int |\psi_n(x)|^4 \mathrm{d}x$$

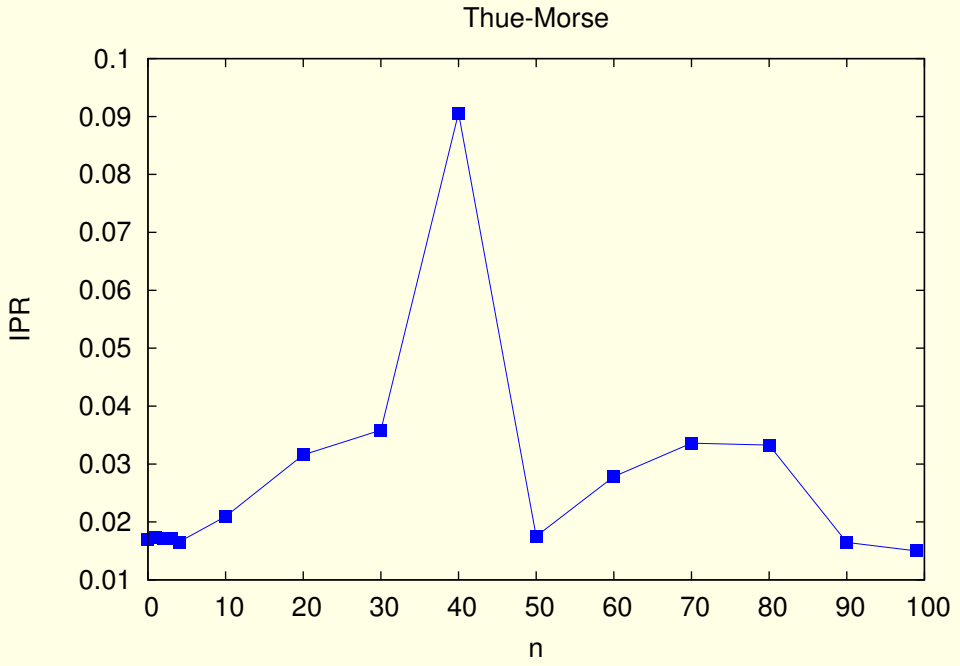





Fibonacci
IPR
n
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60
70
80
90
100


period doubling
IPR
n
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60
70
80
90
100


Rudin-Shapiro
IPR
n
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
50
60
70
80
90
100

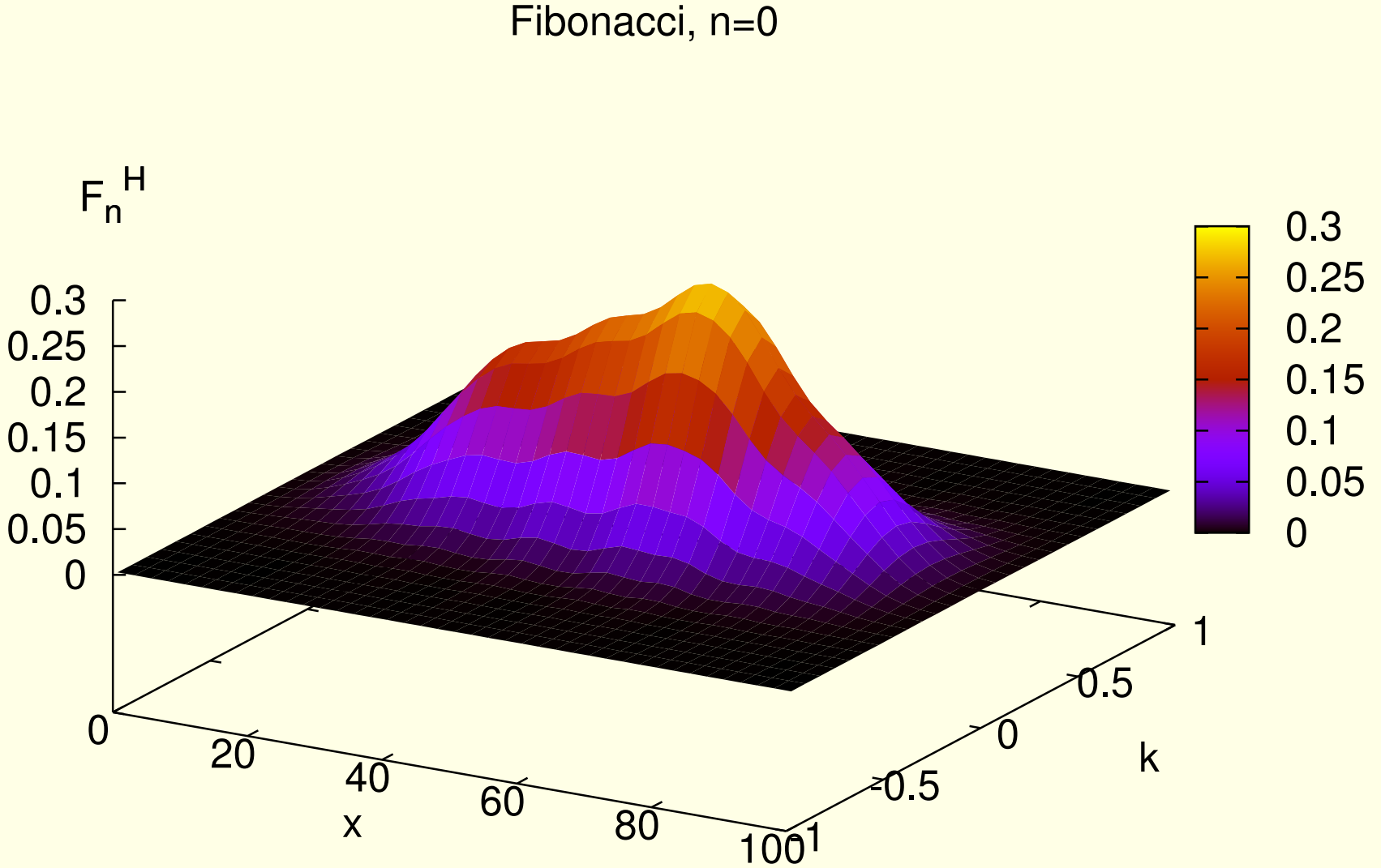

Thue-Morse
IPR
n
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60
70
80
90
100

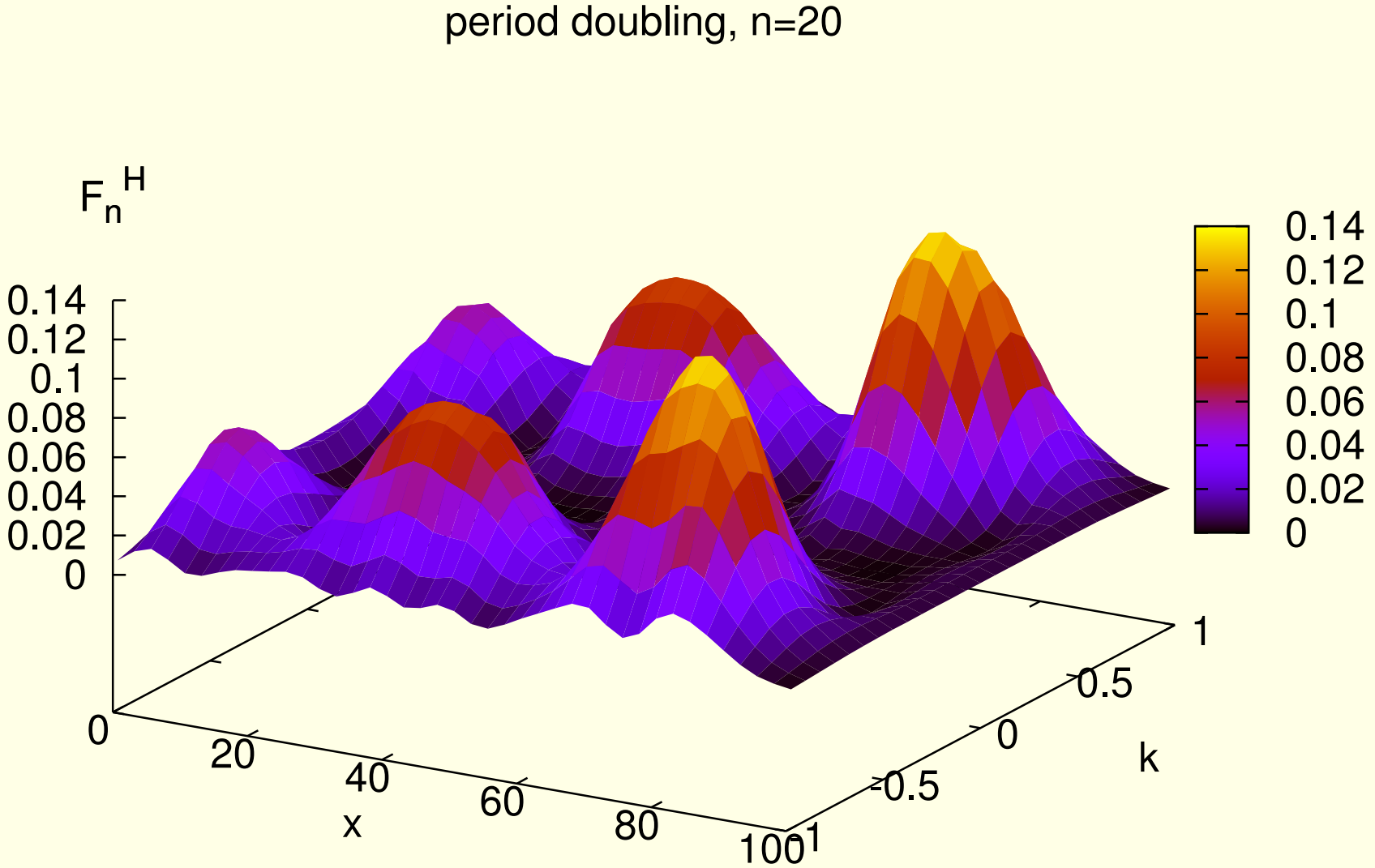
4 Lokalizacja w przestrzeni fazowej

Funkcja rozkładu (funkcja Husimiego)

$$F_n^H(x,k) = \left| \int \mathrm{d}x' \langle x,k|x'\rangle \langle x'|\psi_n\rangle \right|^2 =$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \left| \int \exp\left[-\frac{(x-x')^2}{4\sigma^2} + ikx' \right] \psi_n(x')\,\mathrm{d}x' \right|^2$$


Fibonacci, n=0
F_n^H
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
x
1
0.5
0
-0.5
-1
k
0.3
0.25
0.2
0.15
0.1
0.05
0


period doubling, n=20
F_n^H
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
x
1
0.5
0
-0.5
-1
k

Entropowa długość lokalizacji[a]

$$\lambda_n^W = L \exp(S_n^W - S^{\mathrm{ref}})$$

gdzie

$$S_n^W = -\frac{1}{2\pi} \int F_n^H(x,k) \ln F_n^H(x,k) \, \mathrm{d}x \, \mathrm{d}k$$

$$S^{\mathrm{ref}} = \ln(2L) - 1$$

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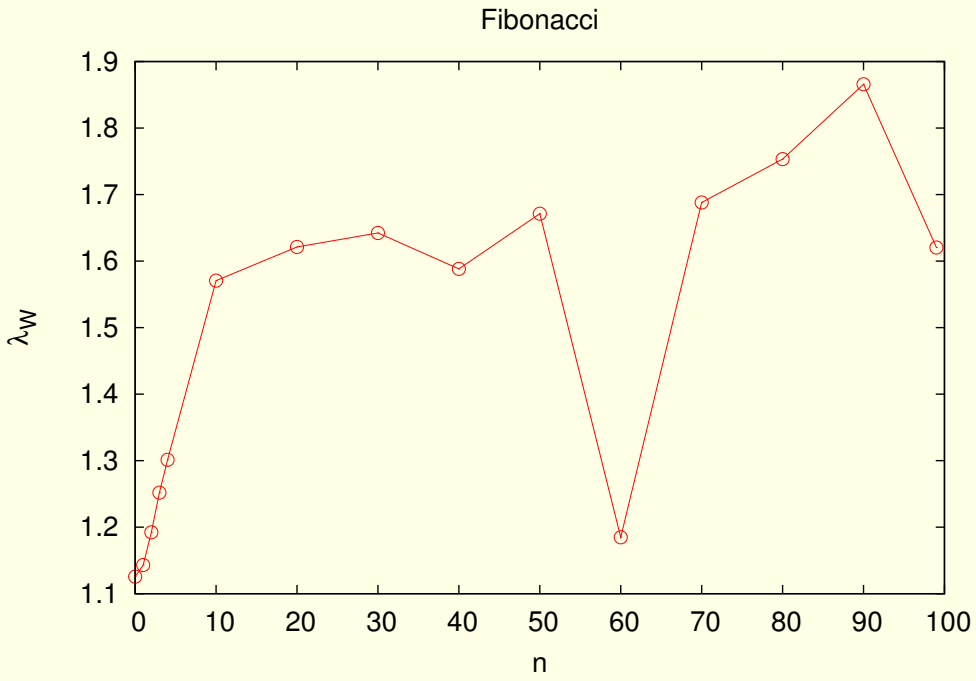
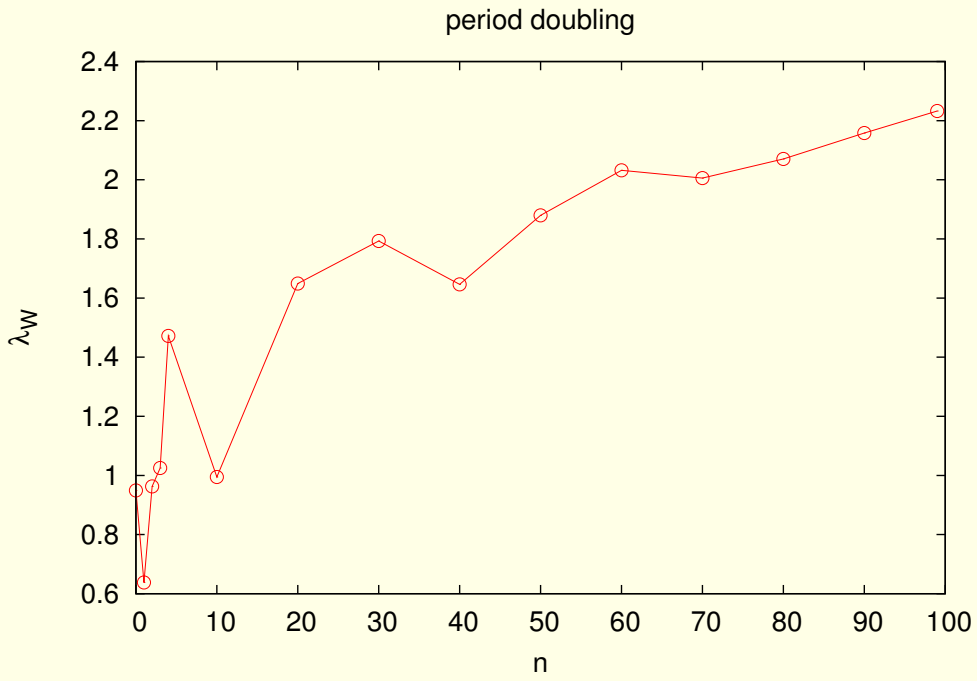
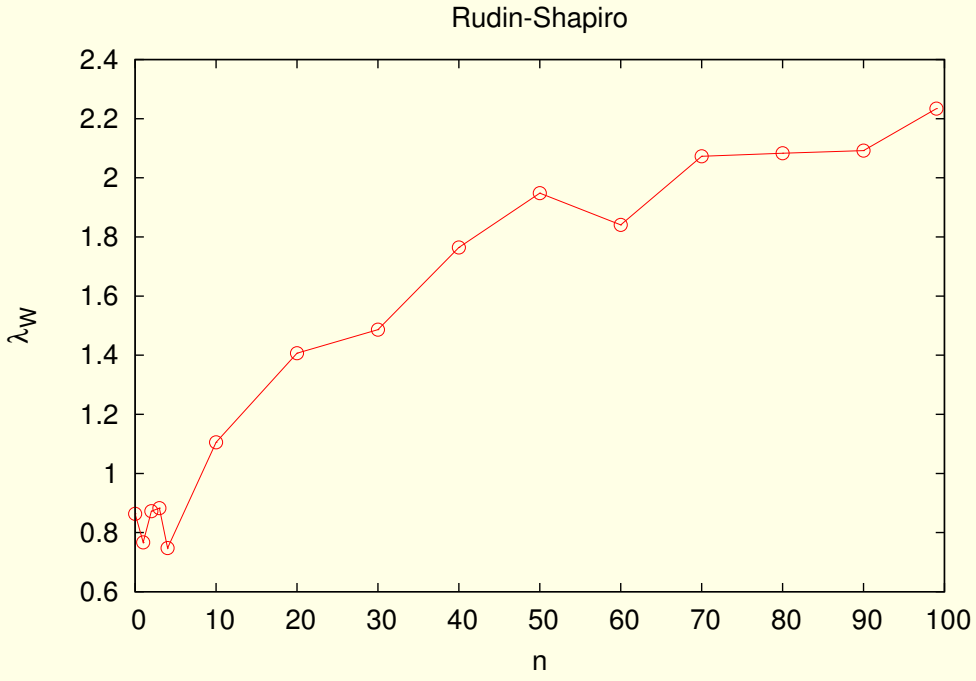
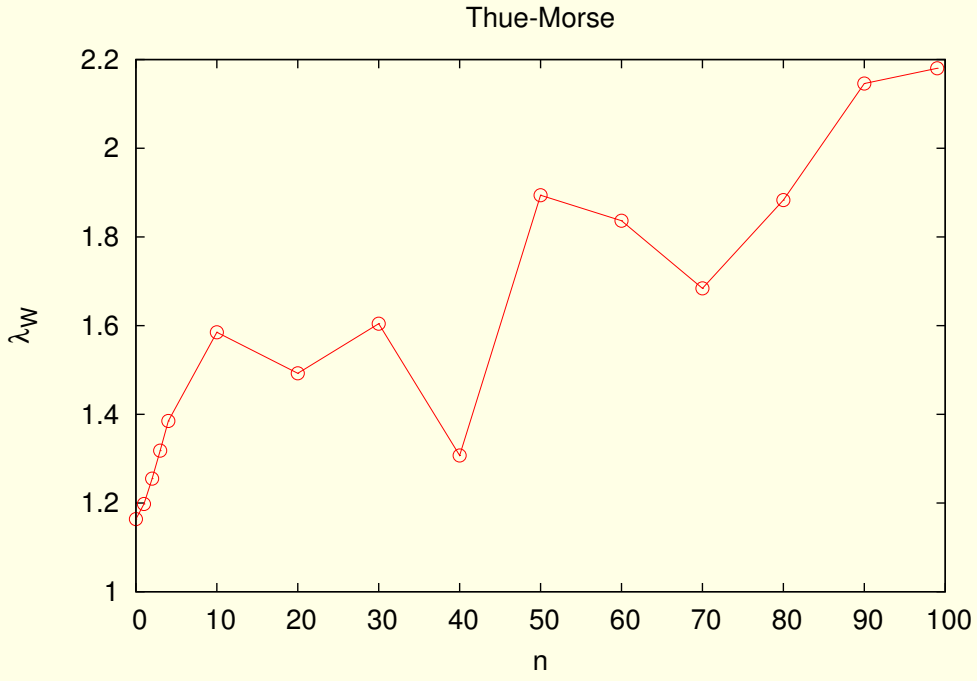
[a] S.Gnutzmann, K.Życzkowski, Renyi-Wehrl entropies as measures of localization in phase space J.Phys.A 34 (2001) 10123


Fibonacci
λW
n
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0
10
20
30
40
50
60
70
80
90
100

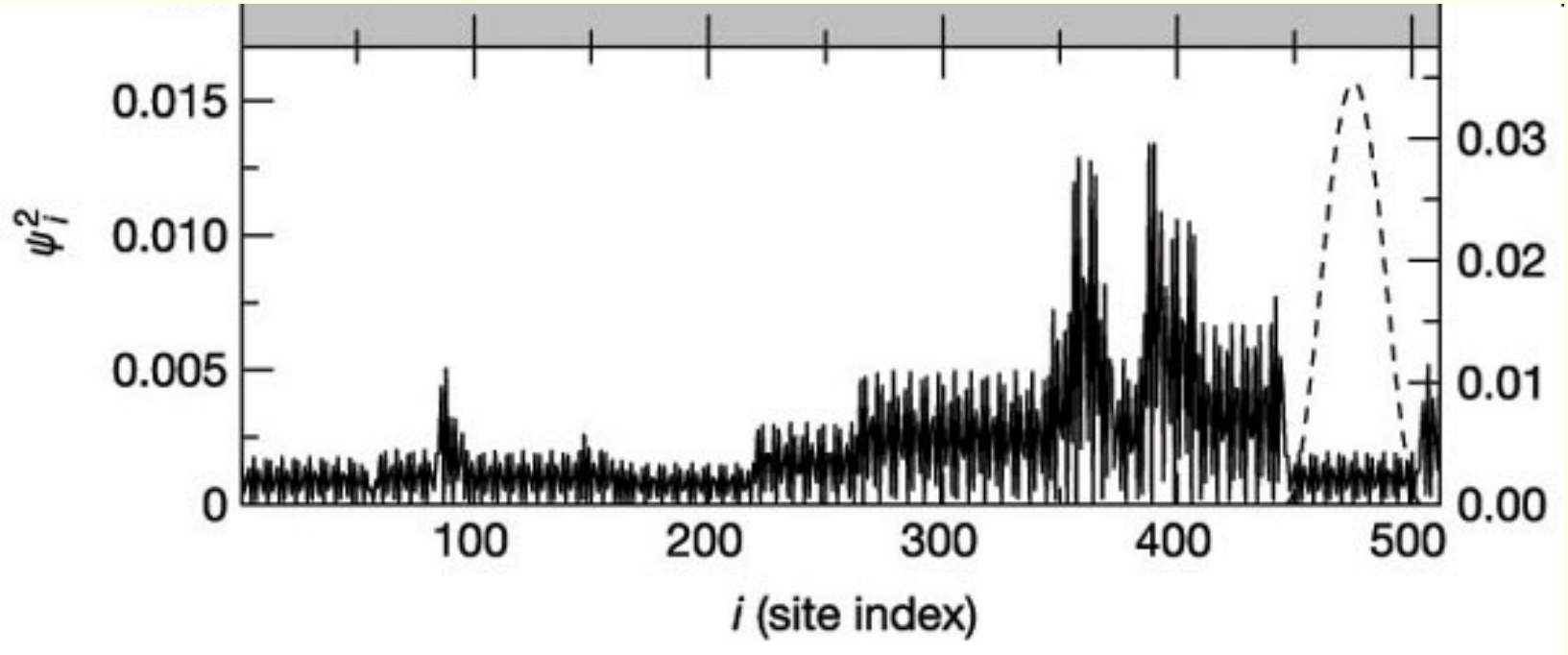

period doubling
λW
n
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
10
20
30
40
50
60
70
80
90
100

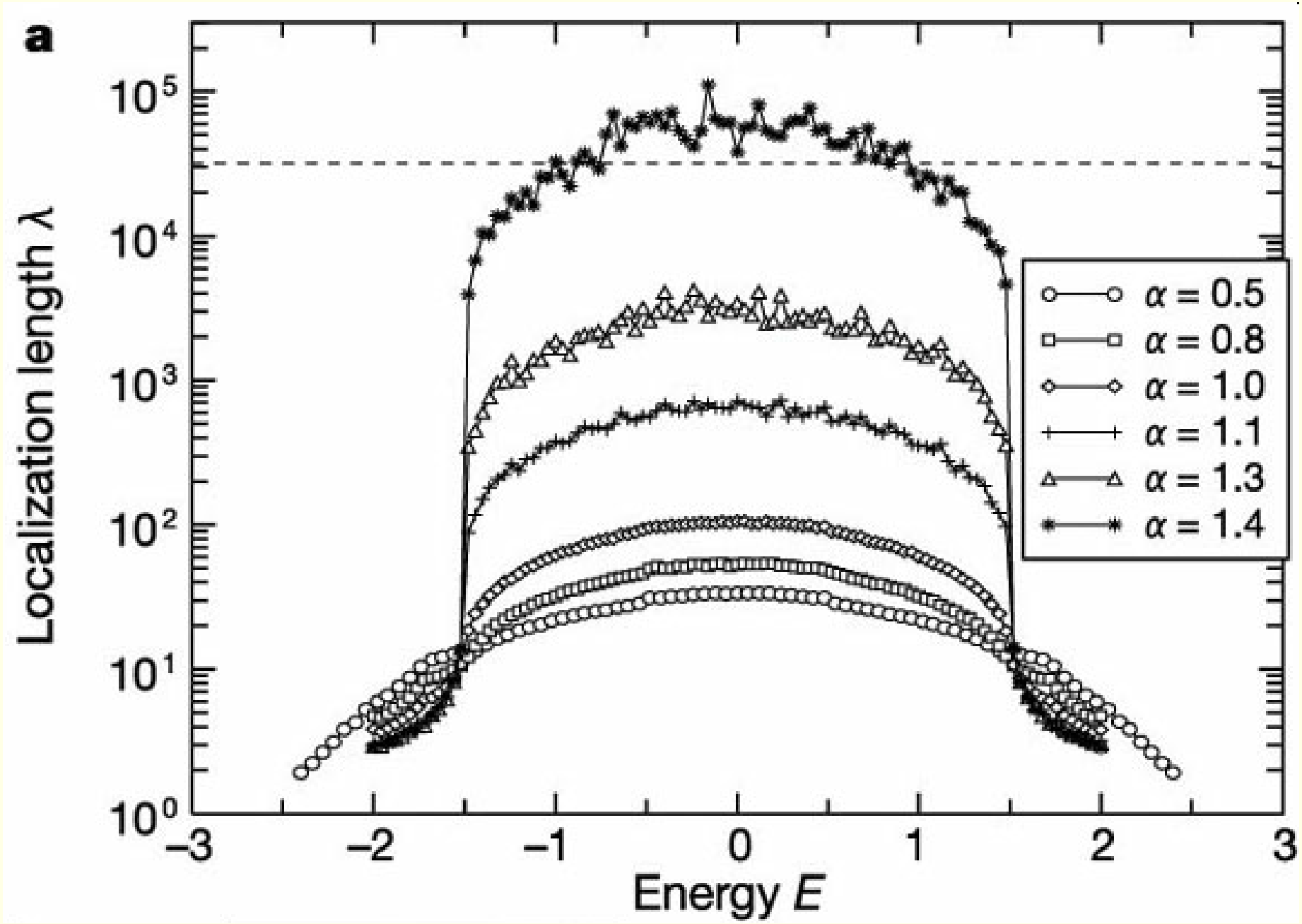

Rudin-Shapiro
λW
n
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
10
20
30
40
50
60
70
80
90
100


Thue-Morse
λW
n
2.2
2
1.8
1.6
1.4
1.2
1
0
10
20
30
40
50
60
70
80
90
100

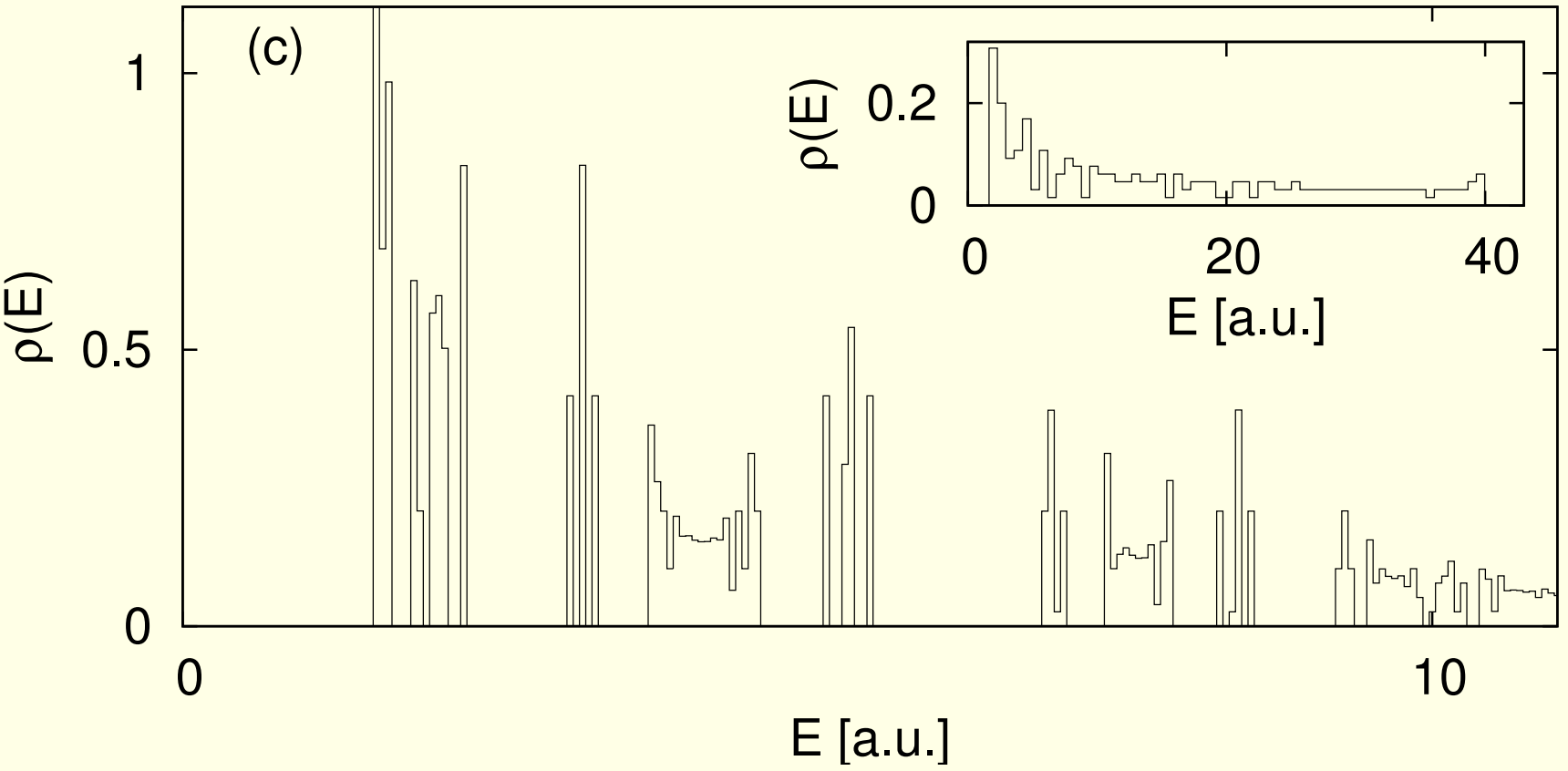
5 Podsumowanie

- P. Carpena et al., Metal–insulator transition in chains with correlated disorder, Nature 418 (2002) 955 $\Longrightarrow$ DNA = 1-wymiarowy nieuporządkowany przewodnik?

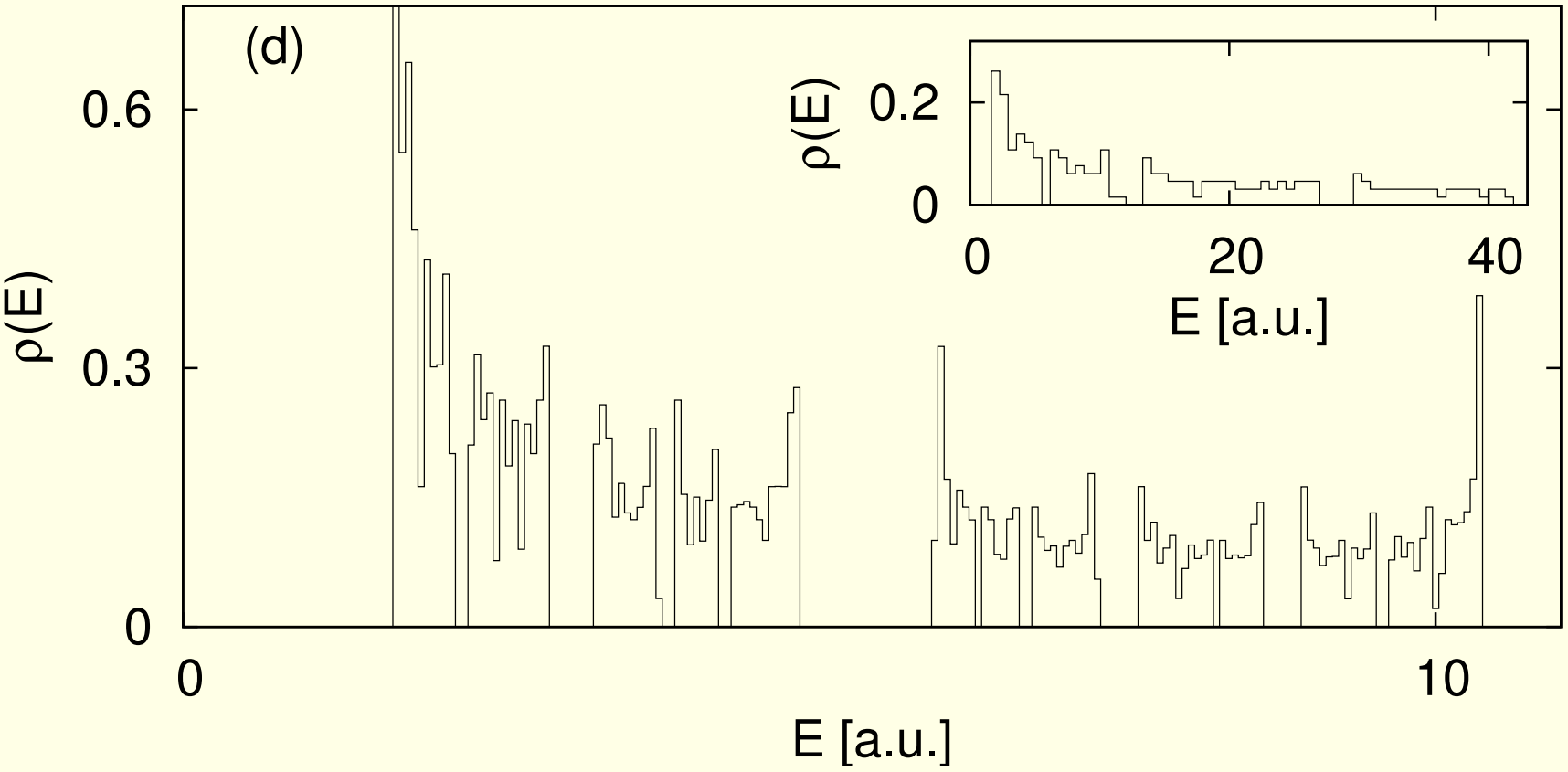



a
Localization length λ
10^5
10^4
10^3
10^2
10^1
10^0
−3
−2
−1
0
1
2
3
Energy E
α = 0.5
α = 0.8
α = 1.0
α = 1.1
α = 1.3
α = 1.4

- znaczenie funkcji gęstości stanów elektronowych



Thue-Morse



Fibonacci