Collective behaviour in clustered social networks

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Motivation

The Strength of Weak Ties by Mark S. Granovetter[1]:

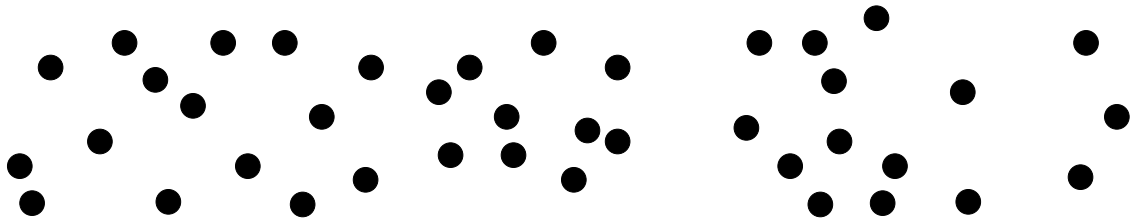
- new information is passed on mainly via distant connections (weak ties) between the groups (cliques),
- community completely partitioned into cliques is usually unable to organise.

[1] M.S. Granovetter, Am. J. of Sociology **78** (1973) 1360

Hierarchy of groups

Model of community proposed by Duncan J. Watts et al.[2]

- N individuals . . .



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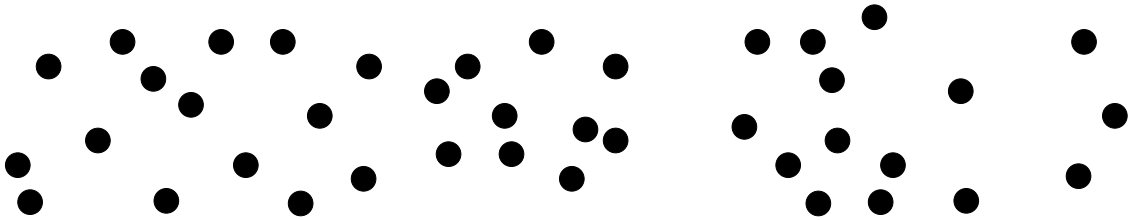
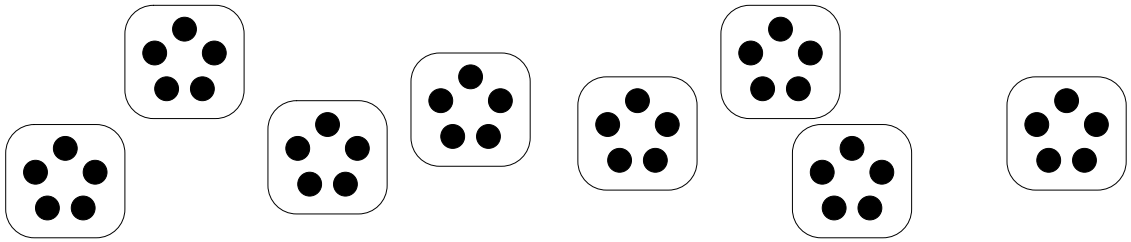


- . . . in small groups (g individuals each) . . .



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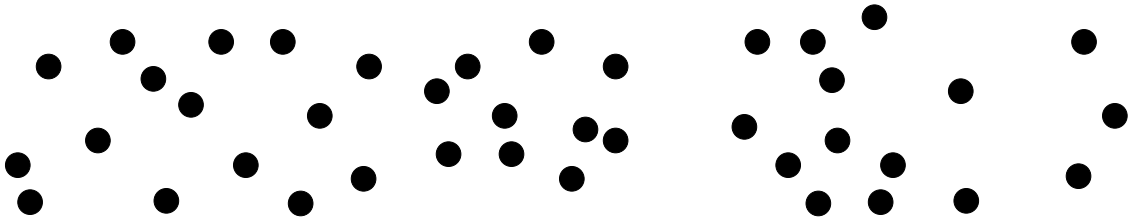

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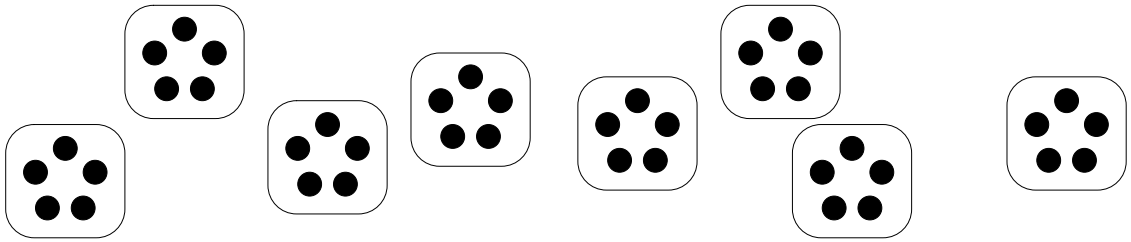
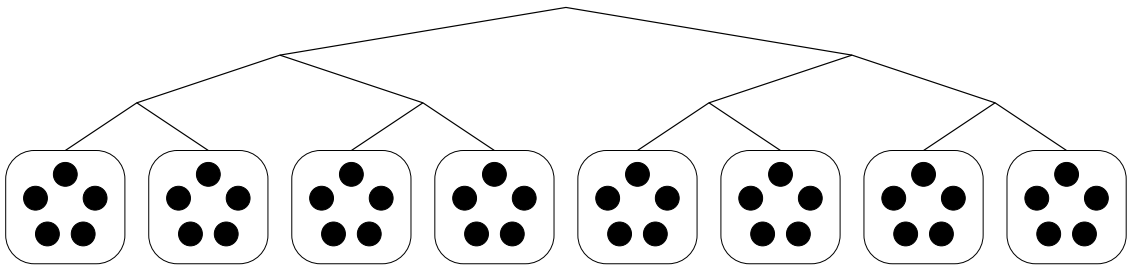


- . . . forming a hierarchy



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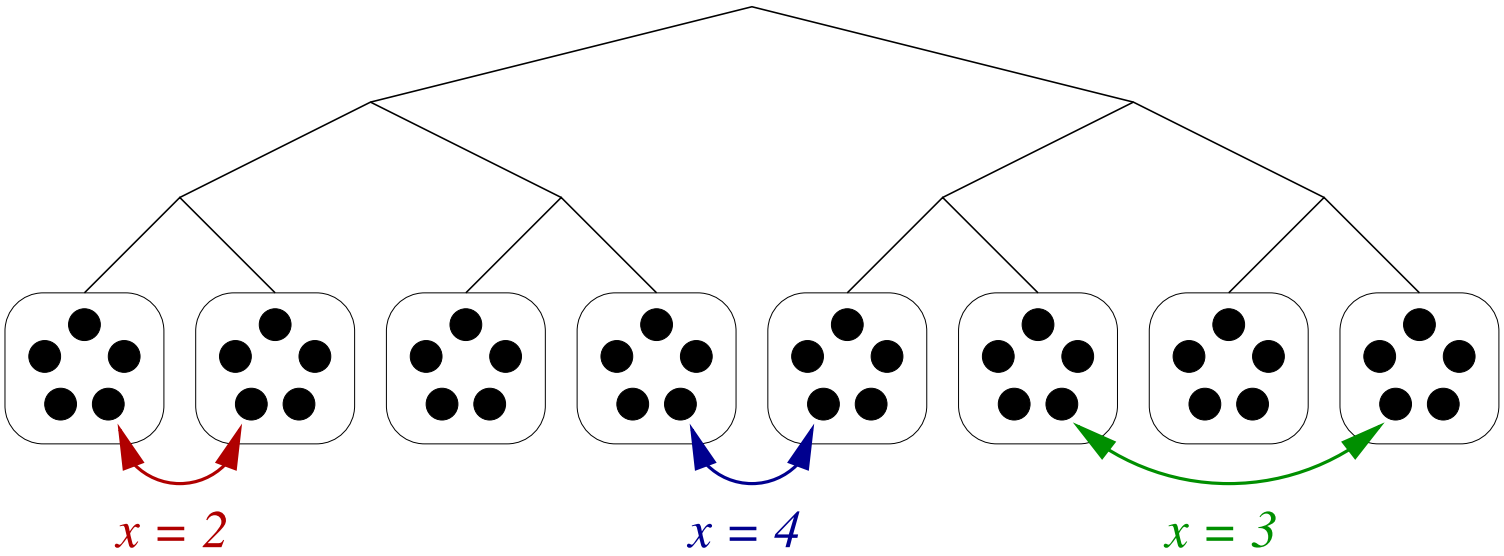

Distance between individuals:

- $x = 1$ for members of the same group,
- $x = 2$ for members of the nearest neighbouring groups,
- $x = 3$ for members of the next nearest neighbouring groups,
- ...

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- link between nodes i and j is established randomly, with probability

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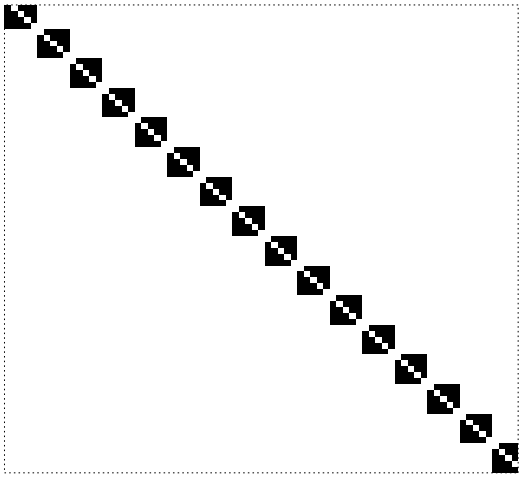
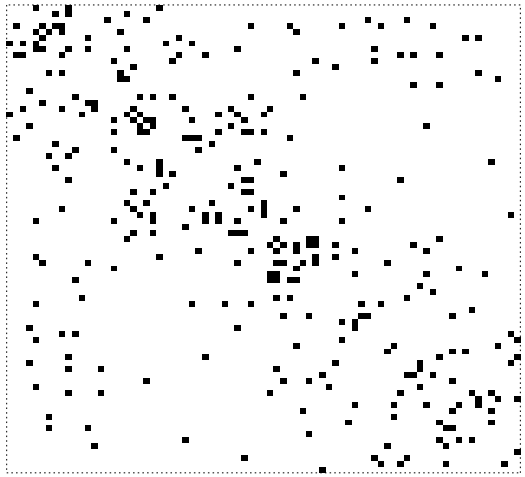
- an *average* number of neighbours $z = g - 1$ is assured

Adjacency matrix (example for $N = 80$, $g = 5$)

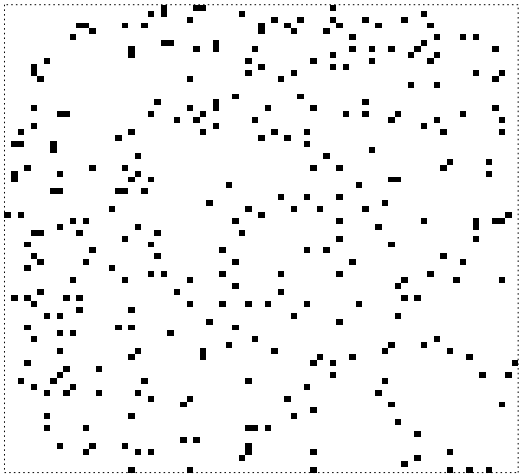
α = 10



α = 0

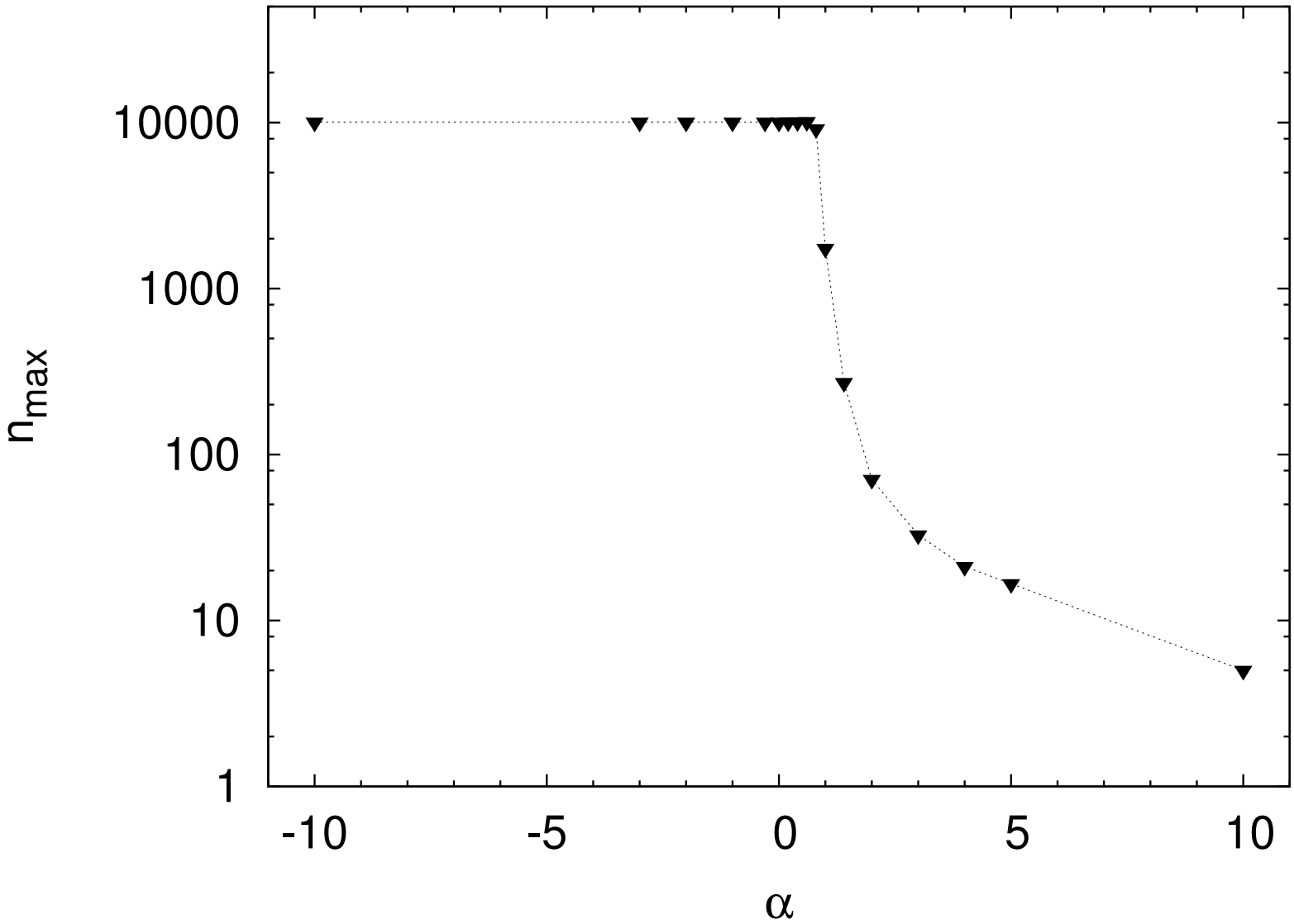


α = - ln 2



α = -10

The largest connected part of the network



Clustering coefficient

- for any given node i

$$C_i = \frac{(\text{links between the vertices within its neighbourhood})}{(\text{number of links that could possibly exist between them})}$$

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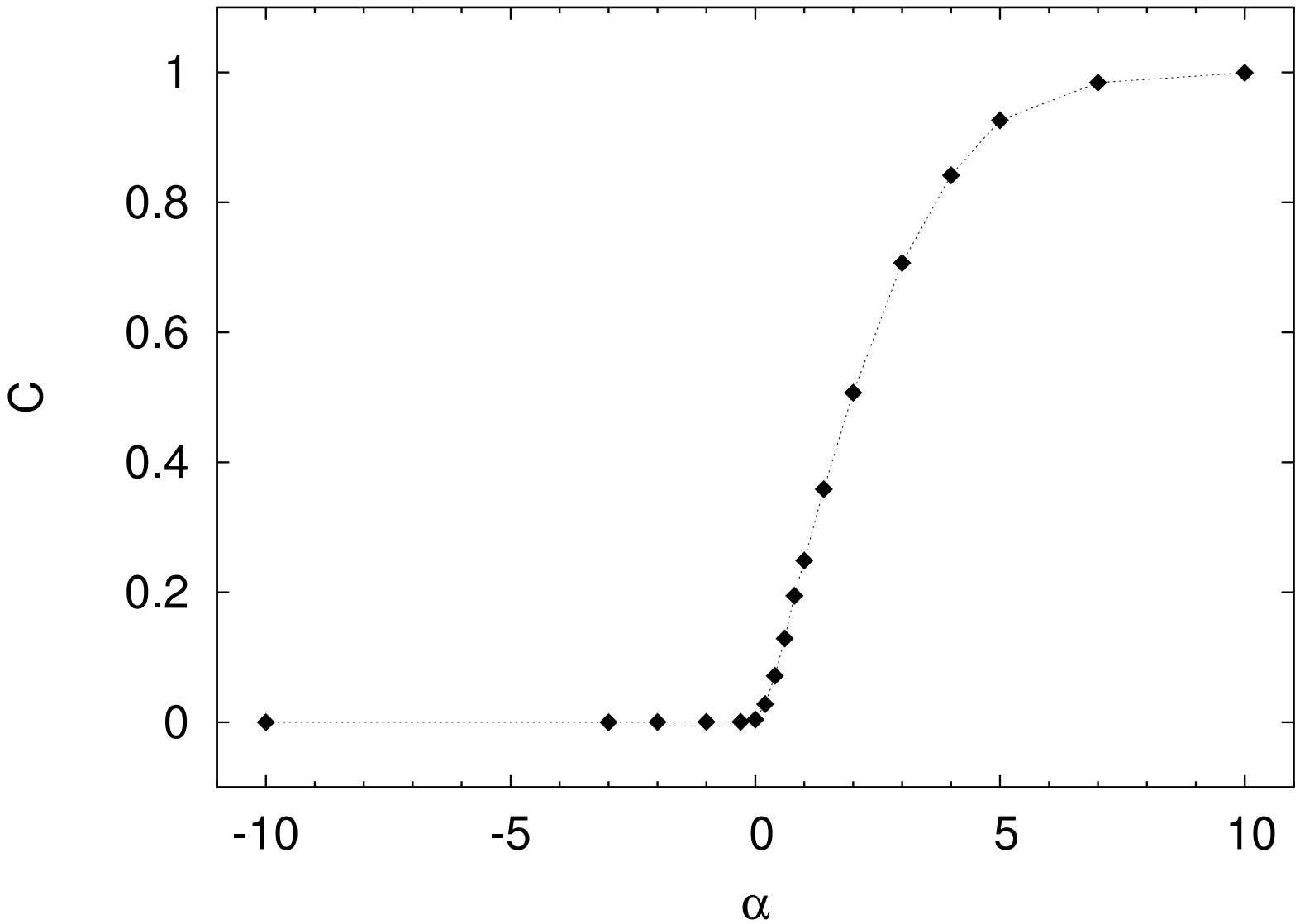
- and for the whole system

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i$$

Clustering coefficient



Model

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- each link $(i, j) \longmapsto$ interaction energy $J_{ij} = J > 0$
  (same opinions preferred for neighbours)

Model

$\Rightarrow$ a magnet with topology similar to Granovetter's suggestion

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- is there any kind of phase transition – ordering of spins ?

  (ability of the social system to collective action ?)

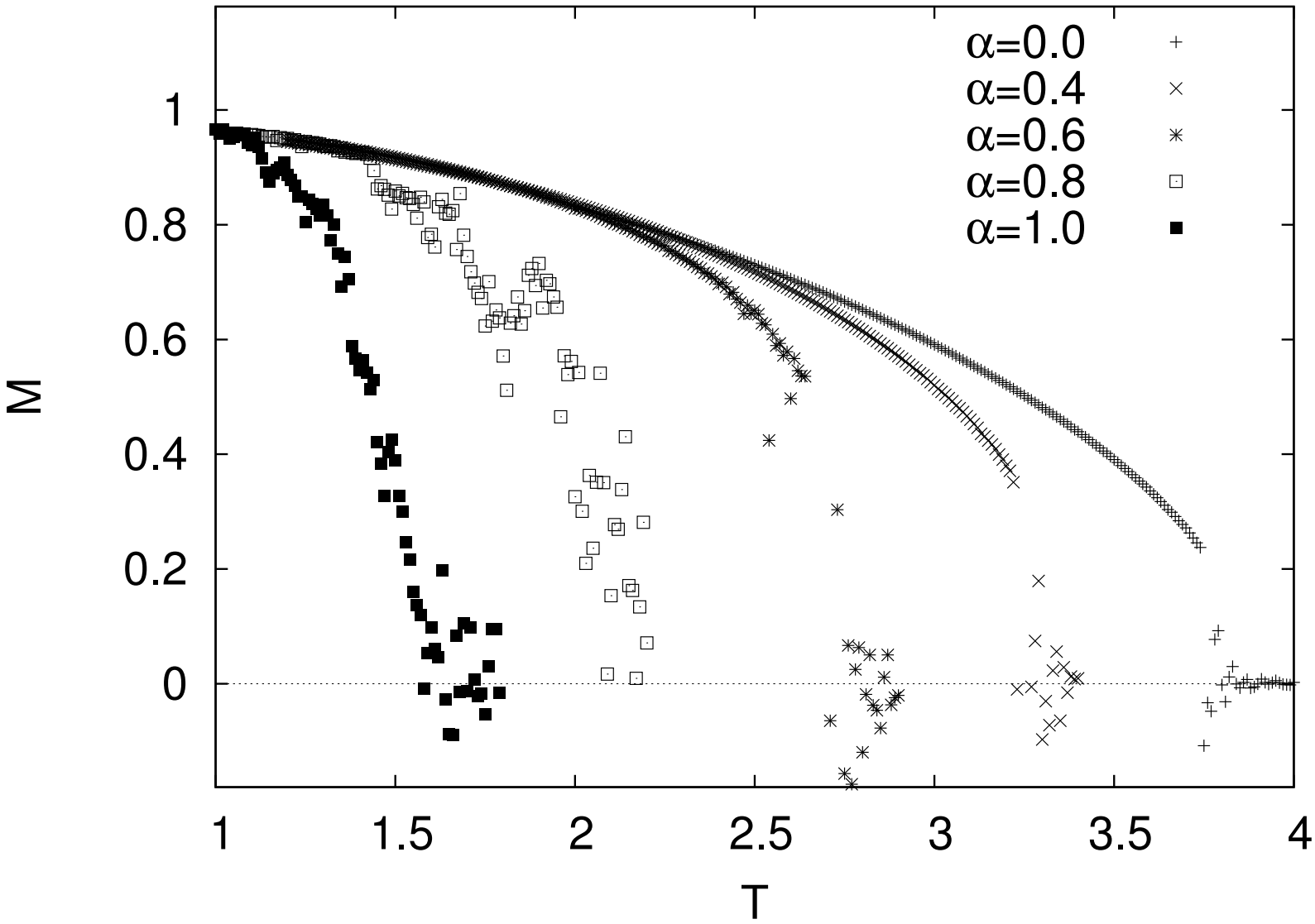
Simulation

- Glauber dynamics – flip probability:

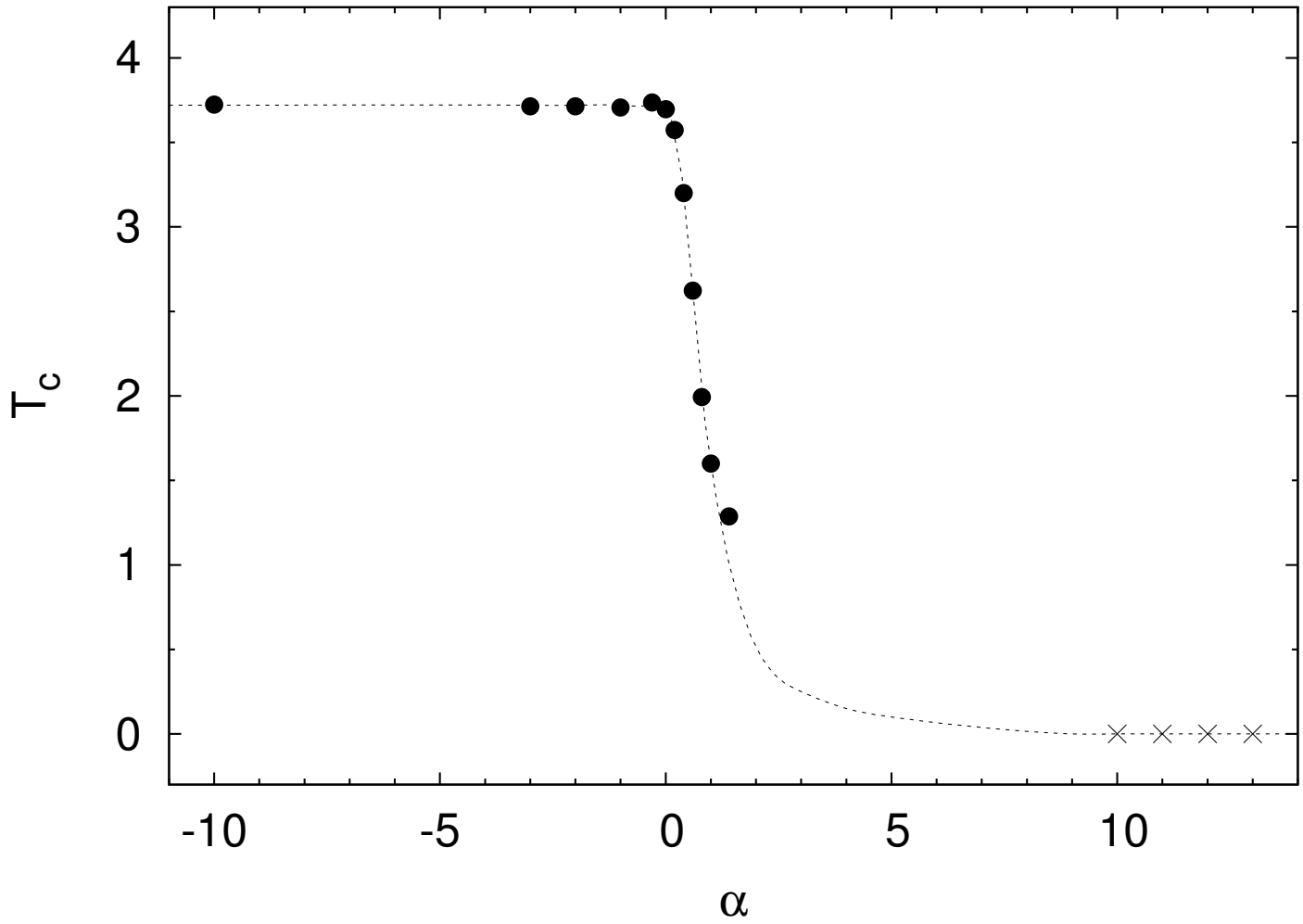
$$p(s_i \to -s_i) = \frac{1}{1 + \exp(\Delta E / kT)}$$

- calculation details:
  - time = 10^5 steps
    (results averaged over last 5×10^4 steps)
  - $N = 10240$
  - $g = 5$

Magnetisation



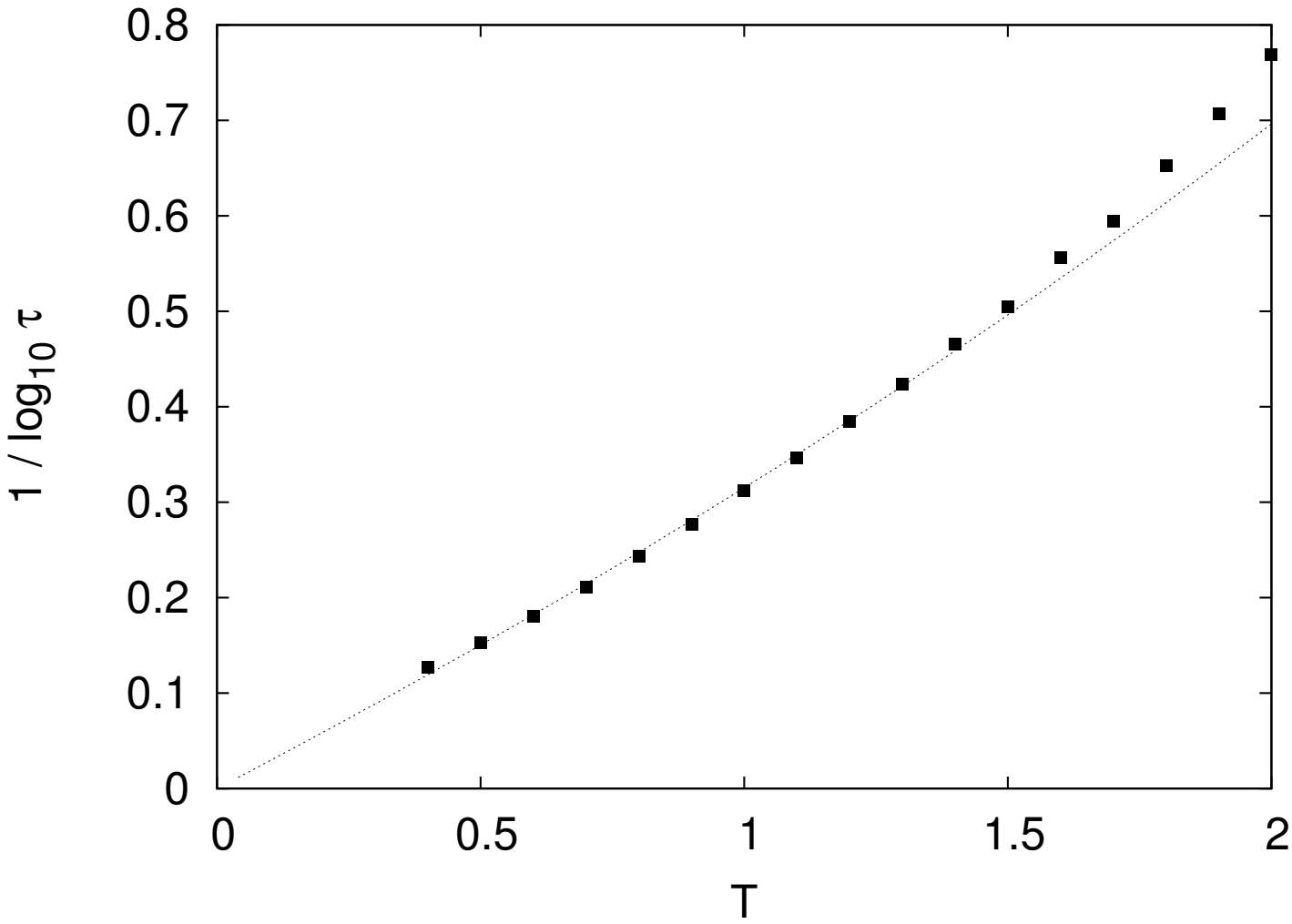
Critical temperature



(averaged over 4 samples)

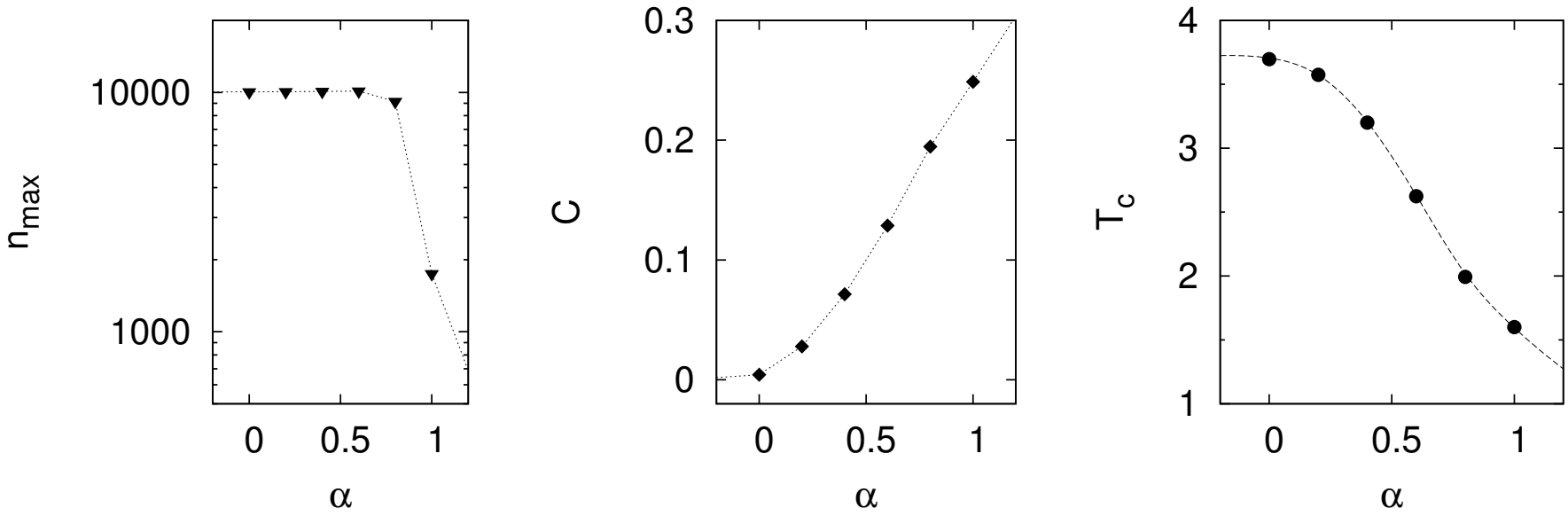
Relaxation time (for $\alpha \geqslant 10$)

- only small, separated groups of $g = 5$ nodes



$0 < \alpha < 0.7$

- ties connecting local groups are weak, but still exist



Conclusions

- there is a critical value of the homophily parameter
- T_c sharply decreases right above $\alpha = 0$ where the network is still connected, but links are too scarce to maintain ordering
- effectiveness of the social interaction depends on the topology of social network: too sparse connections between small groups restrain collective behaviour