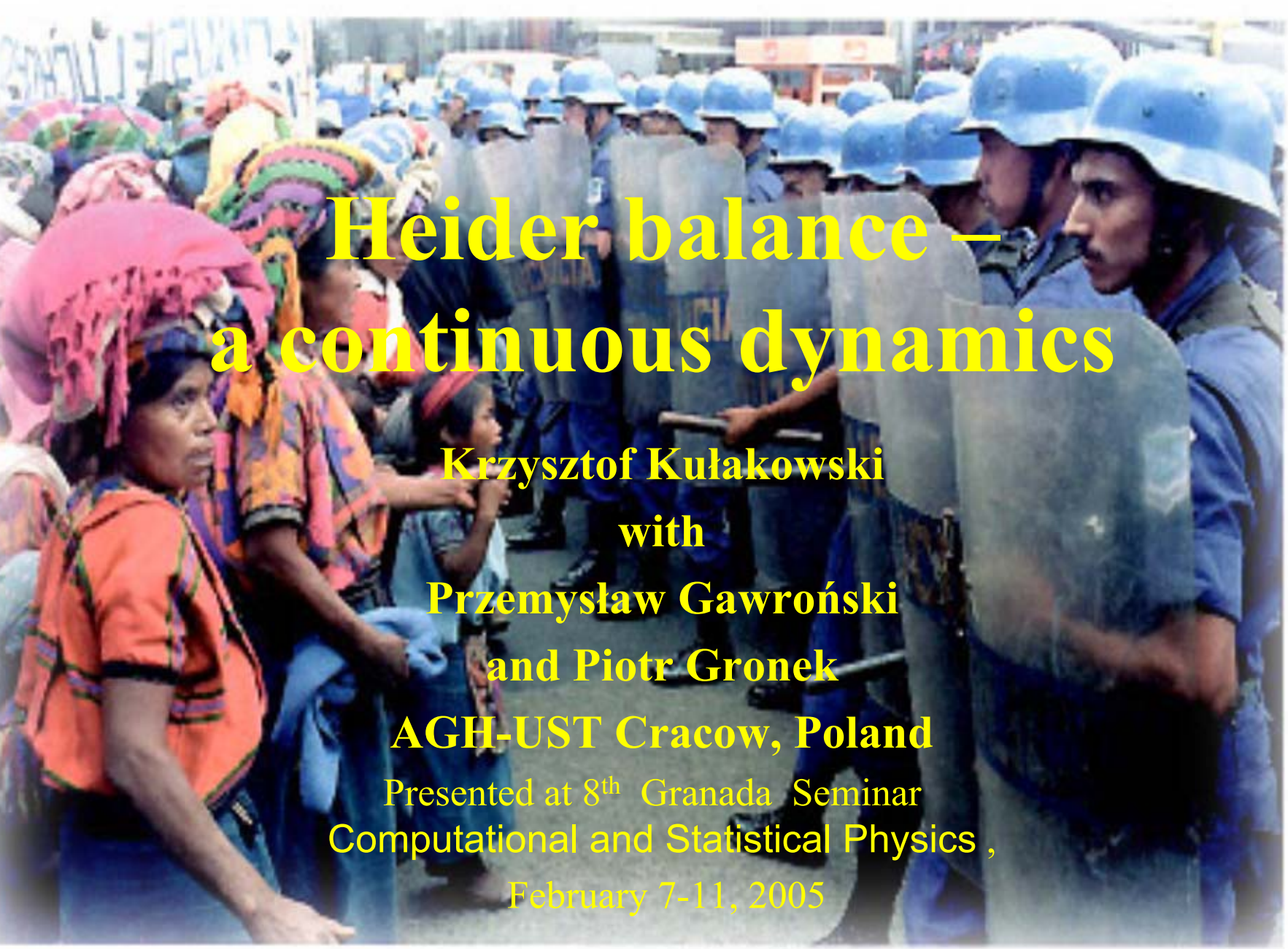

Heider balance – a continuous dynamics

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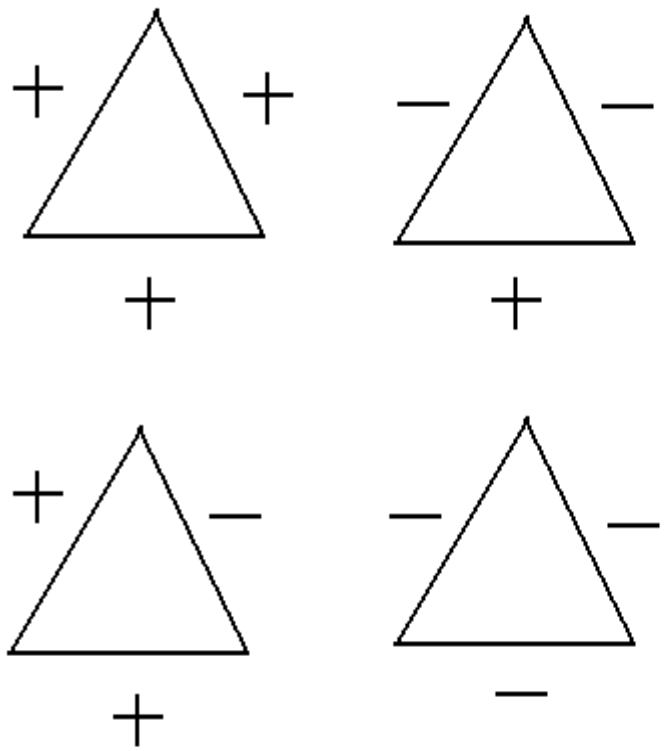
contents

- **What is the Heider balance?**
- **The Bogardus scale**
- **Continuous dynamics**
- **Three examples**
- **Conclusions**

- Friend of my friend is my friend
- Enemy of my friend is my enemy
- Friend of my enemy is my enemy
- Enemy of my enemy is my friend



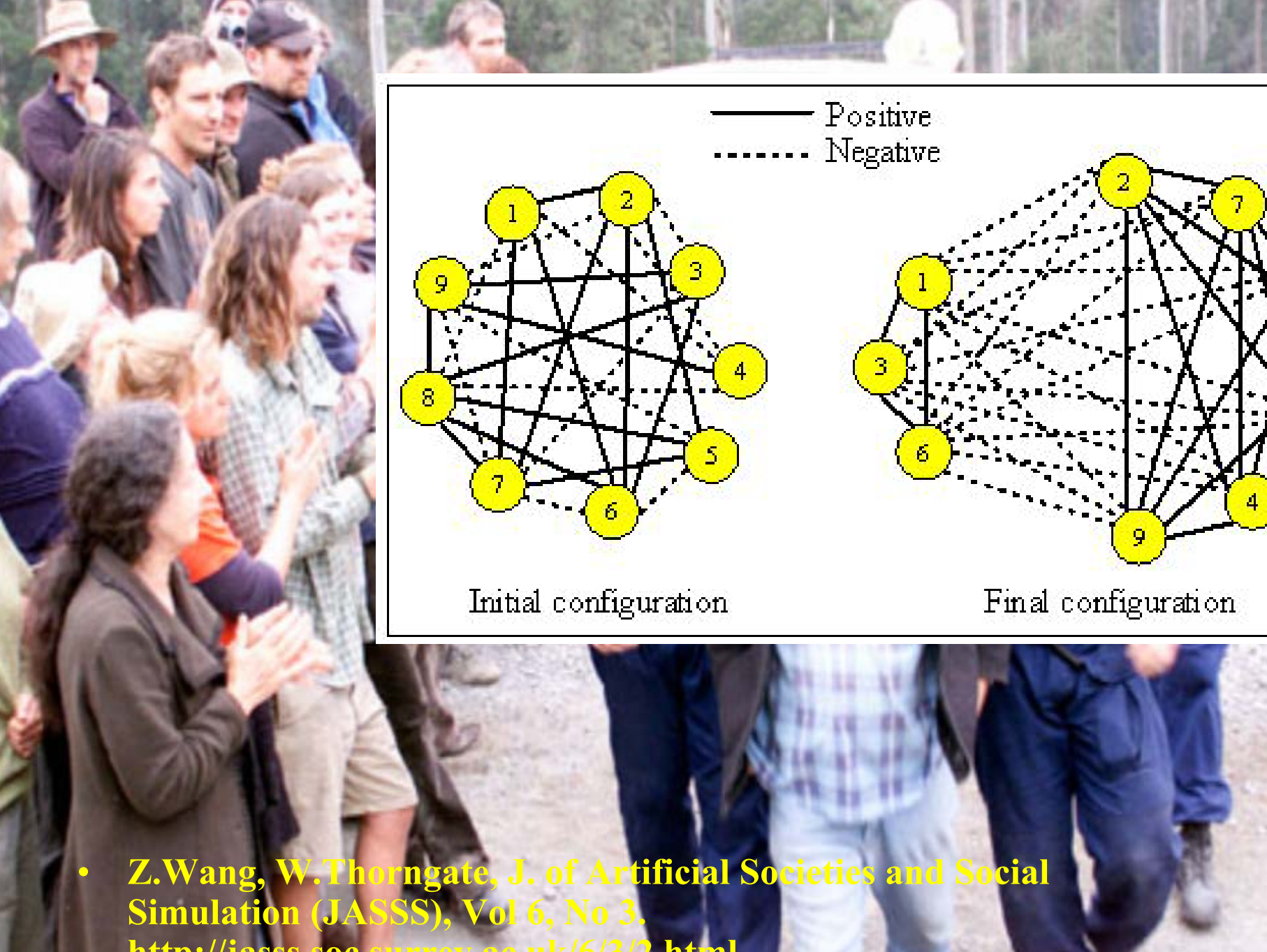
Heider ‘POX’ model (1946)

Heider balance on a network

- **Fully connected graph: each pair of nodes is linked. Each link is represented by $c(i,j) = \pm 1$; (+ is friendship, - is hostility).**
- **In time, unbalanced triads where $c(i,j)c(j,k)c(k,i)<0$ are repaired one by one.**
- **At the end the network is found to be divided in two parts, with friendly relations within them and hostile relations between them.**

  Then the system is balanced.

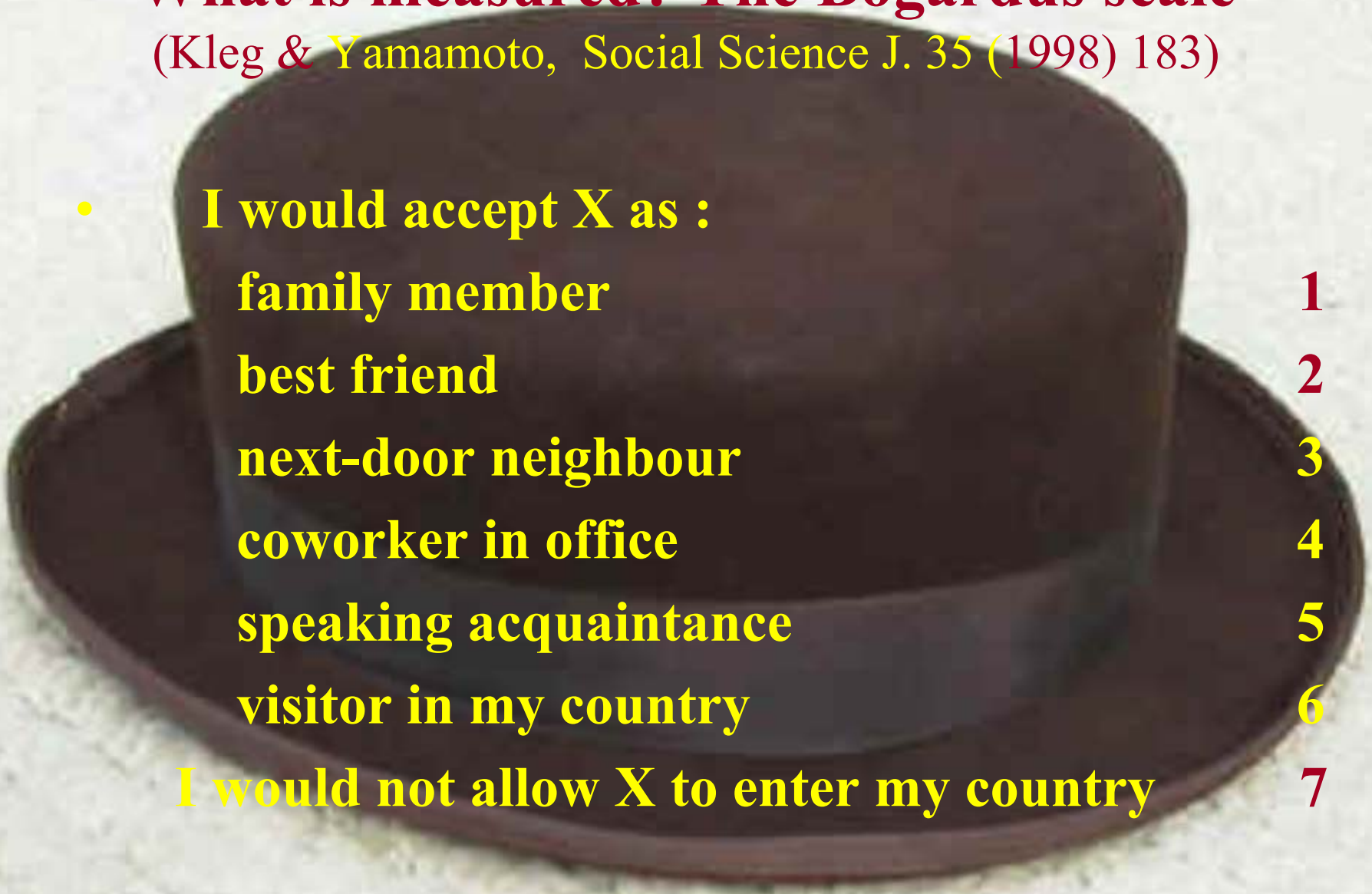
- **Z.Wang, W.Thorngate, J. of Artificial Societies and Social Simulation (JASSS), Vol 6, No 3. http://jasss.soc.surrey.ac.uk/6/3/2.html**



- **Z.Wang, W.Thorngate, J. of Artificial Societies and Social Simulation (JASSS), Vol 6, No 3.**

What is measured? The Bogardus scale

(Kleg & Yamamoto, Social Science J. 35 (1998) 183)



- **I would accept X as :**
  - **family member** **1**
  - **best friend** **2**
  - **next-door neighbour** **3**
  - **coworker in office** **4**
  - **speaking acquaintance** **5**
  - **visitor in my country** **6**

I would not allow X to enter my country **7**

Social distance in USA: means and ranks, 1925 vs 1993

English	1.27	1	1.17	2	Russian	4.57	14	1.33	13
Scottish	1.69	2	1.22	6	Native Amer.	4.65	15	1.44	16
Irish	1.93	3	1.14	1	Jewish	4.83	16	1.42	15
French	2.04	4	1.20	4	Greek	4.89	17	1.38	14
Dutch	2.12	5	1.25	9	Arab	5.00	18	2.21	24
Swedish	2.44	6	1.21	5	Mexican	5.02	19	1.56	18
Danish	2.48	7	1.23	7	Black Amer.	5.10	20	1.55	17
Norwegian	2.67	8	1.25	8	Chinese	5.28	21	1.68	20
German	2.89	9	1.27	10	Japanese	5.30	22	1.62	19
Spanish	3.28	10	1.29	11	Korean	5.55	23	1.72	21
Italian	3.98	11	1.19	3	Turk	5.80	24	1.77	22
Hindu	4.35	12	1.95	23					
Polish	4.57	13	1.30	12	Grand Mean	3.82		1.43	

(Kleg & Yamamoto, Social Science J. 35 (1998) 183)

NEW: a continuous dynamics

Instead *c(i,j)*=±1 we use real numbers *r(i,j)*

Equation of motion:

$$\frac{dr(i,j)}{dt} = \sum_k r(i,k)r(k,j)$$

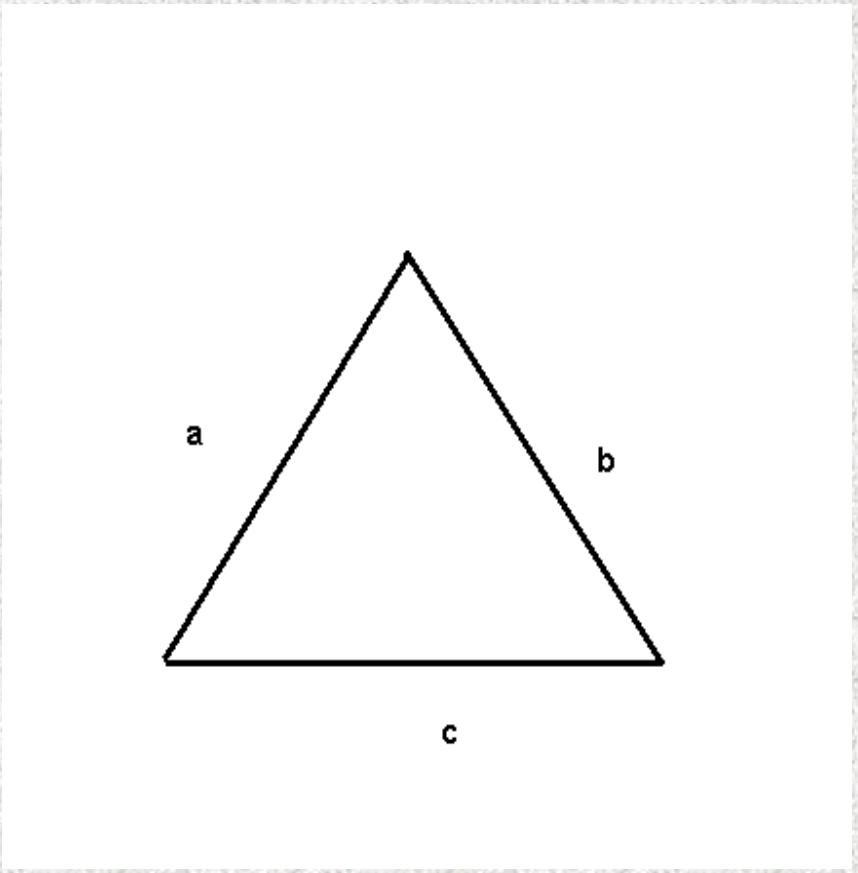
or, with limited range of *r(i,j)*:

$$\frac{dr(i,j)}{dt} = \left[1 - \frac{r^2(i,j)}{R^2}\right] * \sum_k r(i,k)r(k,j)$$

where $-R < r(i,j) < R$.

These equation lead to the Heider balance as well.

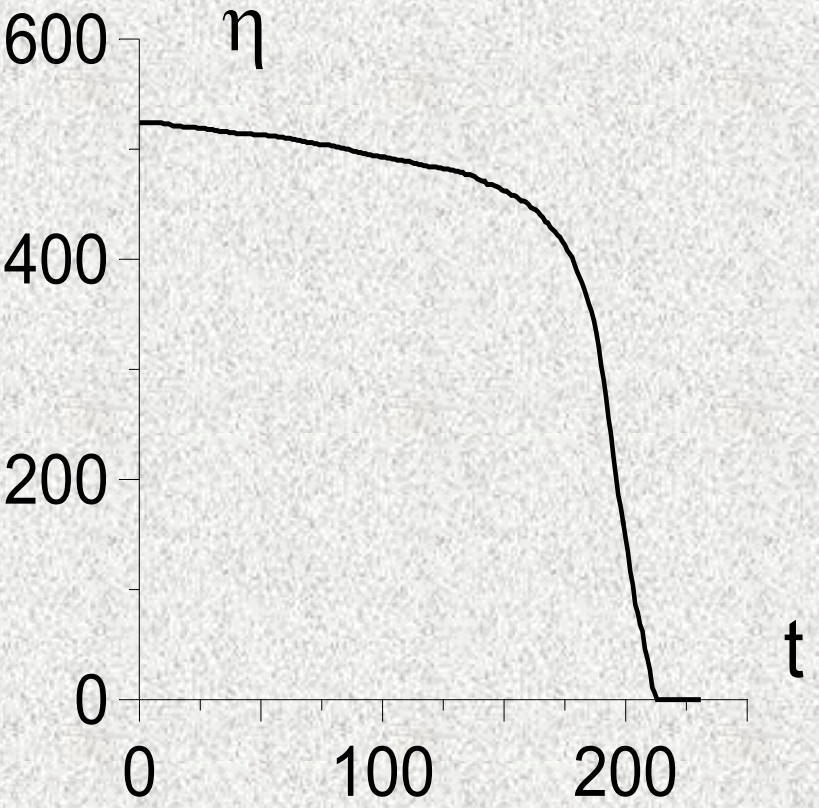
Example: N=3

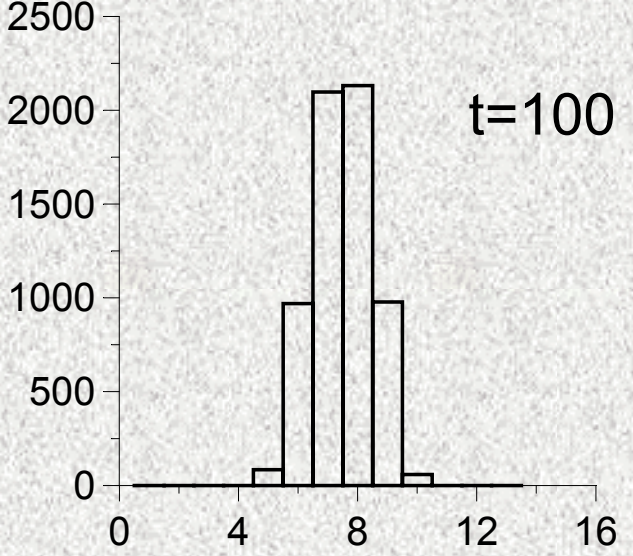


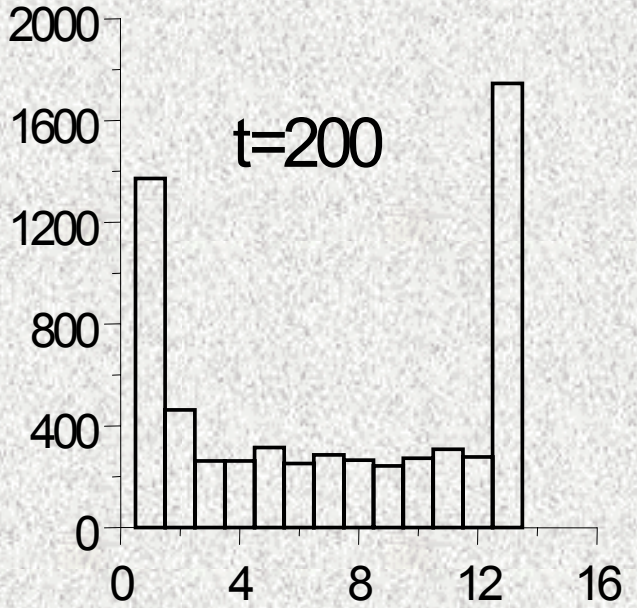
$$da/dt = b\ c\ (1-a^2/R^2)$$
$$db/dt = c\ a\ (1-b^2/R^2)$$
$$dc/dt = a\ b\ (1-c^2/R^2)$$

- The balance condition $abc>0$ implies, that either $a>0$ and a increases or $a<0$ and a decreases, and the same for b, c.

Dynamics of the distribution of r(i,j).

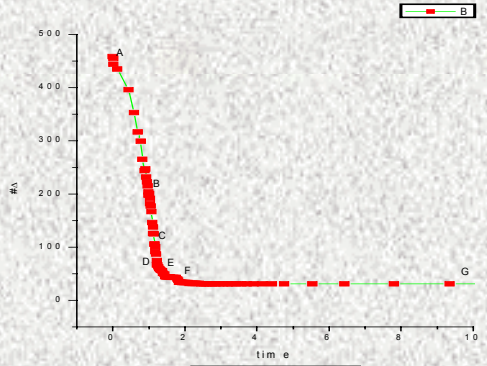


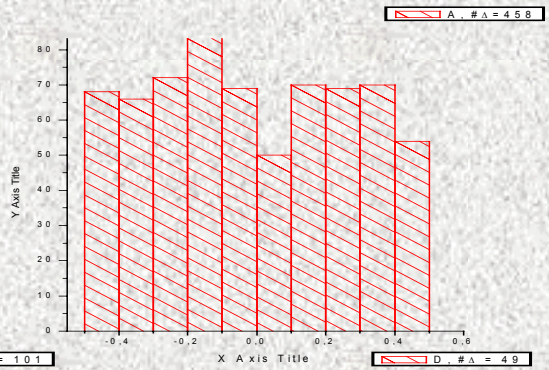


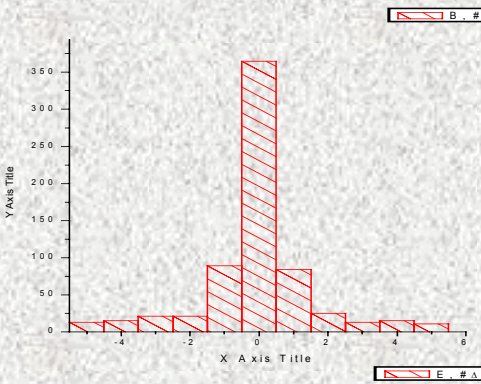


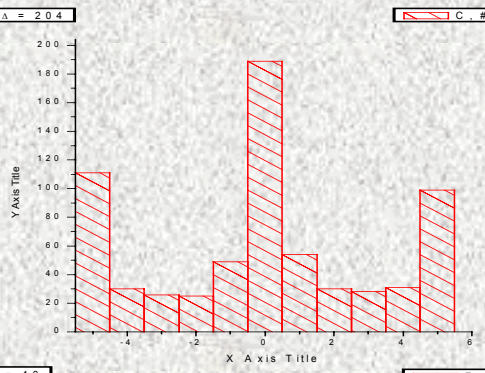
Polarization of opinions: an example when the balance is NOT reached

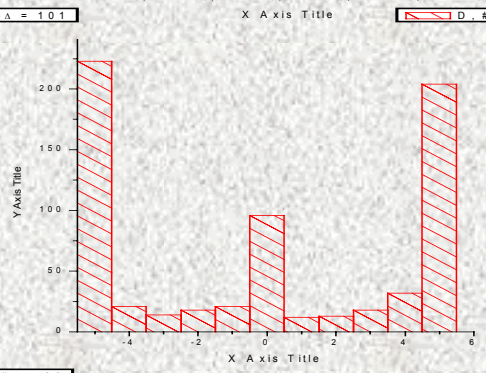
AB, M=7

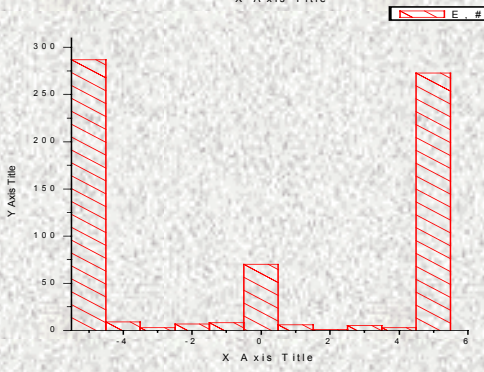


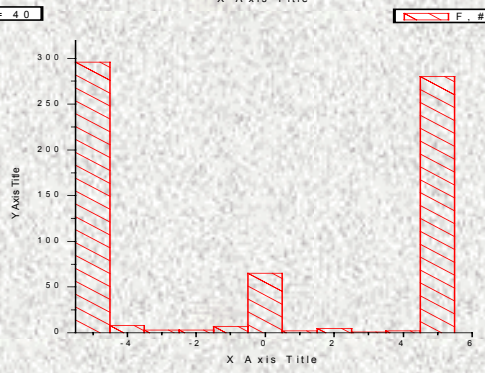


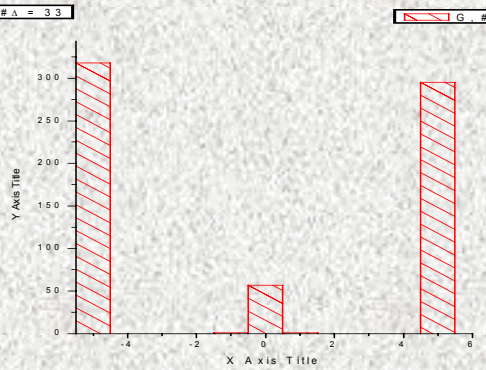




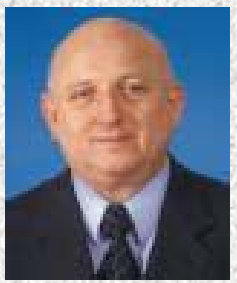




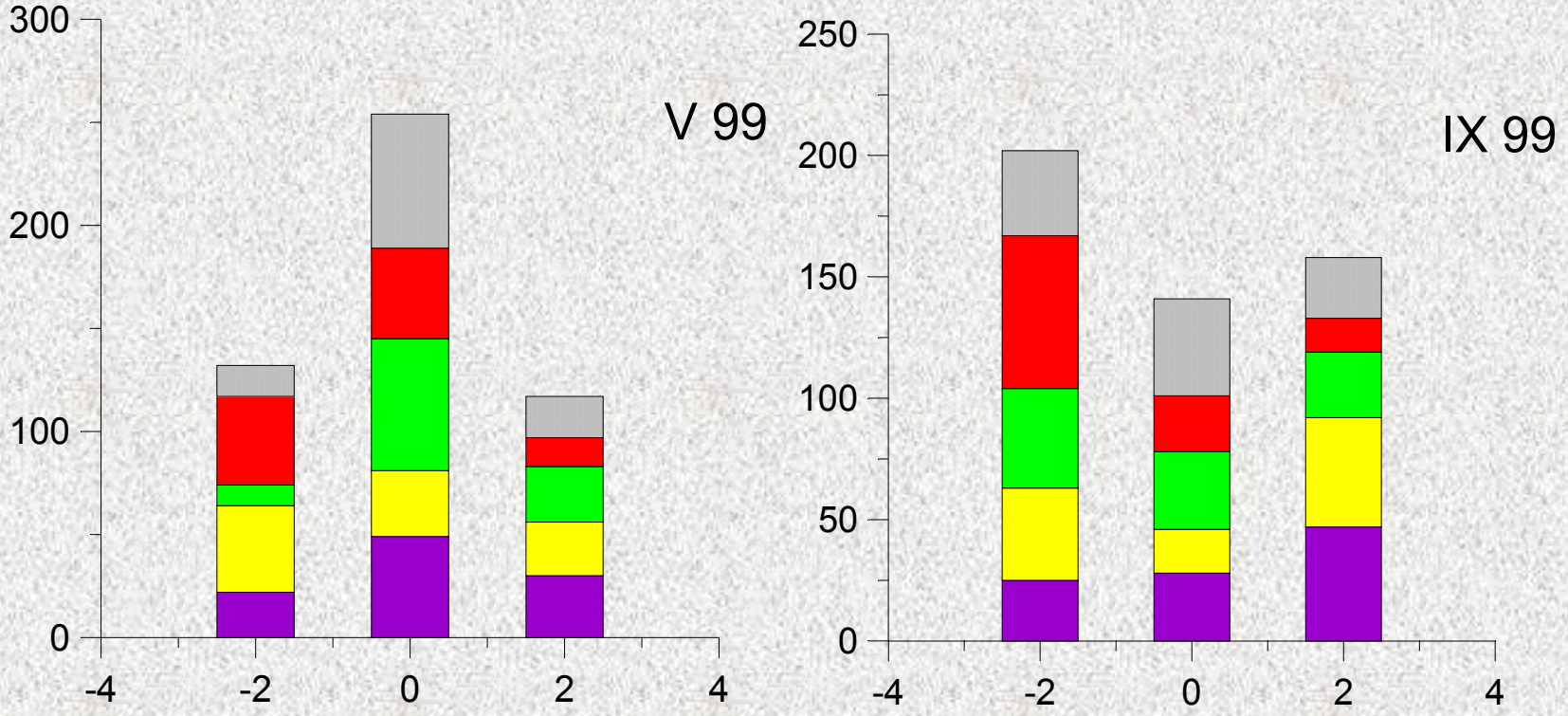




Example 1: Polish politics, 1999



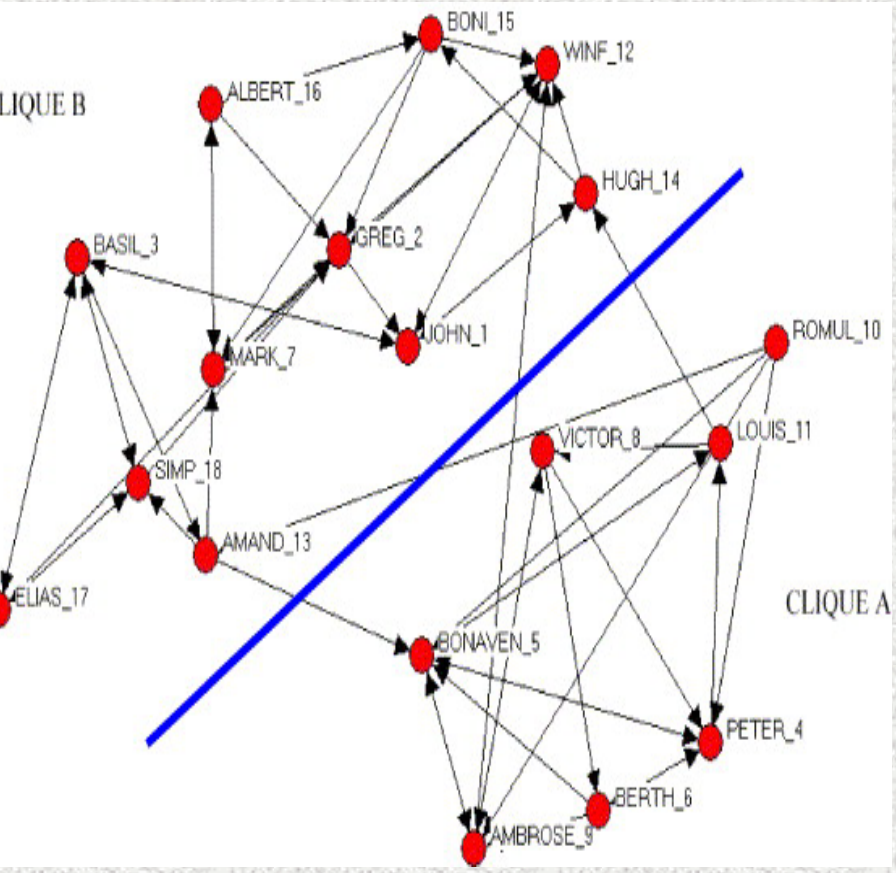
Enquiry: How you evaluate the effects of the lustration law?



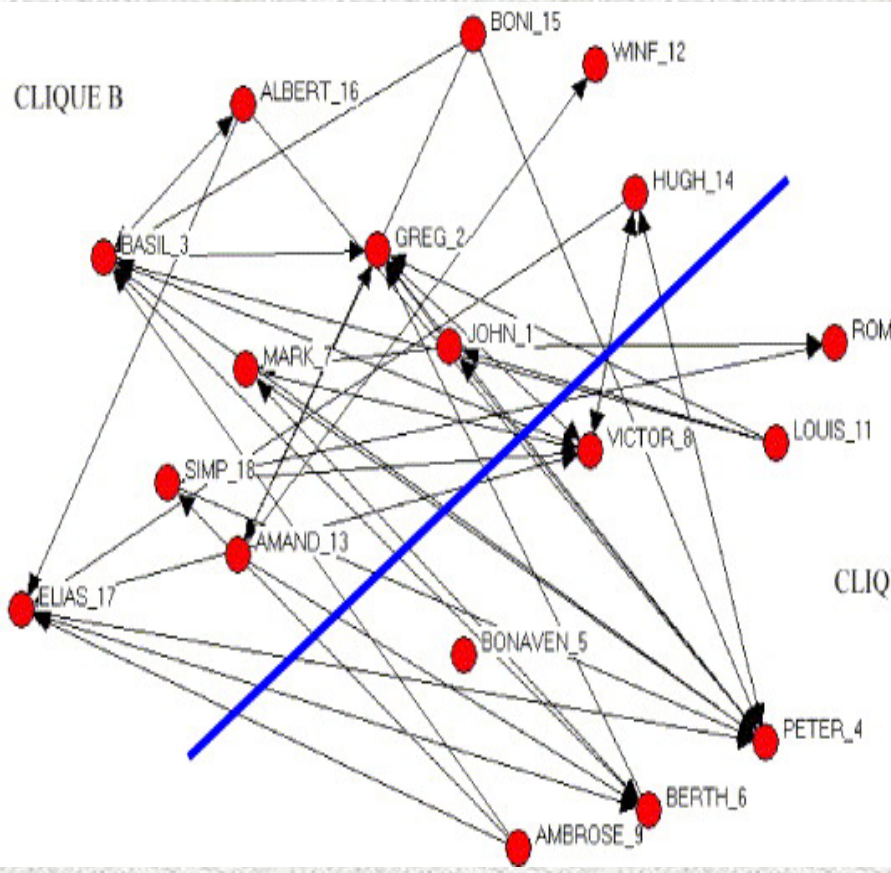
Details: report of CBOS, BS/152/1999 (in Polish)

Example 2: a Monastery in England, 1968

I like



I dislike



Sampson, S. PhD Thesis, Cornell 1969
(Ph. Bonacich, P.Lloyd, Social Networks 26 (2004) 331)

Example 2 – continued:

THE DATA: **'I like X(3),Y(2),Z(1)' at times t1, t2, t3,**
'I dislike X(-3),Y(-2),Z(-1)' at time t4
(details in Doreian+Mrvar, Soc.Net. 18 (1996) 149).

GRAPHICAL METHOD: (Bonacich+Lloyd, Soc.Net. 26 (2004) 331)
input from t1+t4: no balance, no dynamics,
4,5,6,8,9,10,11 ***vs*** **1,2,3,7,12,13,14,15,16,17,18**

OUR SIMULATION:
input from t1+t4: the balance occured after some time,
1,2,4,5,6,8,9,10,11,14,15,16 ***vs*** **3,7,12,13,17,18**
The results are stable vs a symmetric noise.

REAL LIFE: **2,3,17,18 expulsed;**
1,7,14,15,16 left soon,
4,8,10,12,13 left gradually,
5,6,9,11 remained.

Example3 : Southern Women, 1935

Names of Participants of Group I	Code Numbers and Dates of Social Events Reported in *Old City Herald*													
	(1) 6/27	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 8/3
1. Mrs. Evelyn Jefferson	X	X	X	X	X	X		X	X					
2. Miss Laura Mandeville	X	X	X		X	X	X	X						
3. Miss Theresa Anderson		X	X	X	X	X	X	X	X					
4. Miss Brenda Rogers	X		X	X	X	X	X	X						
5. Miss Charlotte McDowd			X	X	X		X							
6. Miss Frances Anderson			X		X	X		X						
7. Miss Eleanor Nye					X	X	X	X						
8. Miss Pearl Oglethorpe						X		X	X					
9. Miss Ruth DeSand					X		X	X	X					
10. Miss Verne Sanderson							X	X	X			X		
11. Miss Myra Liddell								X	X	X		X		
12. Miss Katherine Rogers								X	X	X		X	X	X
13. Mrs. Sylvia Avondale							X	X	X	X		X	X	X
14. Mrs. Nora Fayette						X	X		X	X	X	X	X	X
15. Mrs. Helen Lloyd							X	X		X	X	X		
16. Mrs. Dorothy Murchison								X	X					
17. Mrs. Olivia Carleton									X		X			
18. Mrs. Flora Price									X		X			

Example 3: continued

hot. by Ben Shahn, Natchez, MS, October 1935

- **L.C. Freeman, 2003 : comparison of 21 methods of finding social groups. Result: rank of methods, from 0.543 to 0.968, based on mutual accordance**
- **Here: the correlation function *r(i,j)=P(i/j)P(j)-P(i)P(j)*. Result: 0.968.**

Ph.Bonacich+P.Lloyd, 2004 : how to find status of group members, if *r(i,j)<*

We applied the simulation to find a hypothetical time dependence of the status. The division remained unchanged in the time evolution.

The cliques are: 1-9, 10-18. The leaders are: Laura and Brenda in clique A, Catherine and Nora in clique B.

Conclusions

- **For the fully connected networks, our continuous dynamics leads to the Heider balance as well as the integer dynamics of Wang and Thorngate. However, the continuous dynamics is more rich and more appropriate from the sociological point of view.**

- **A direct consequence of the continuous dynamics is the opinion polarization. It is observed also in other networks, where Heider balance is not always reached. This polarization can be measured and used for predicting conflicts.**

- More results on fully connected graphs:
  KK, PG, P.Gronek, IJMPC 2005, physics/0501073;
  KK, PG, P.Gronek, submitted, physics/0501160
  More results on the Albert-Barabasi networks: **OUR POSTER HERE**